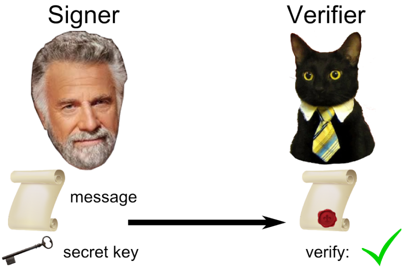
The Fiat–Shamir Transformation in a Quantum World

Özgür Dagdelen Marc Fischlin Tommaso Gagliardoni

CASED and EC-SPRIDE and TU Darmstadt

Asiacrypt'13, December 4th, 2013
Bengaluru, India

Signature scheme



Signer

Verifier

message

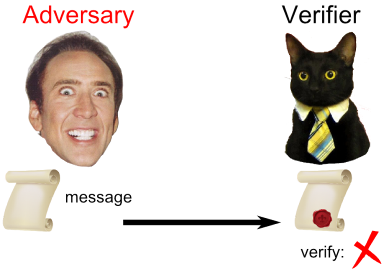
secret key

verify: ✓

Signature scheme

Security:

no efficient adversary can successfully forge a valid signature without knowing the secret key



Adversary

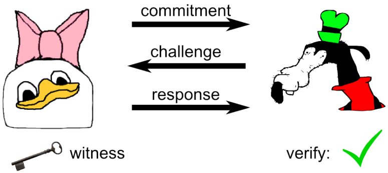
Verifier

message

verify: ✗

Identification scheme

Prover Verifier



commitment

challenge

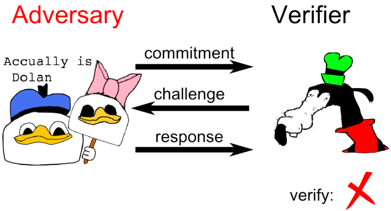
response

witness

verify: ✓

statement: "I am Daisy"

Identification scheme



Adversary

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challenge

response

verify: ✗

statement: "I am Daisy"

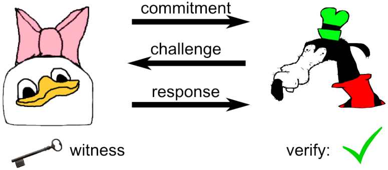
Security:

no efficient adversary can successfully prove identity without valid witness

Identification scheme

Prover

Verifier



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Signature scheme

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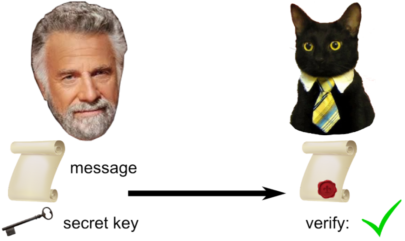


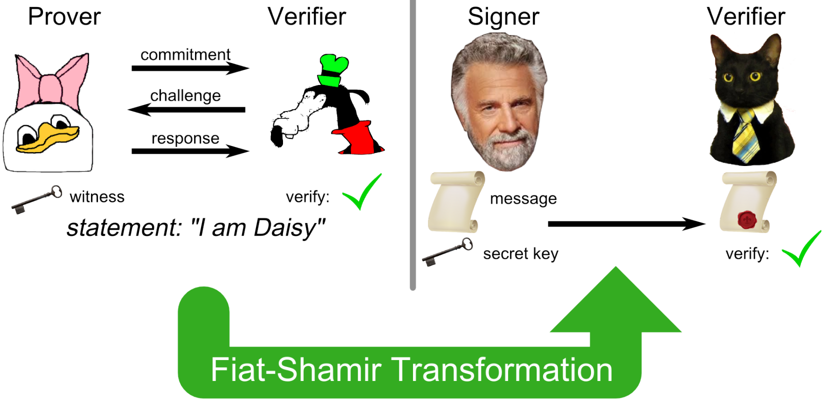
message

secret key

verify: ✓

Identification scheme

Signature scheme



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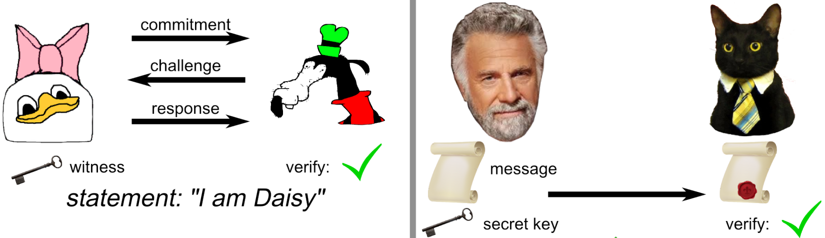
Fiat-Shamir Transformation

Identification scheme

Signature scheme



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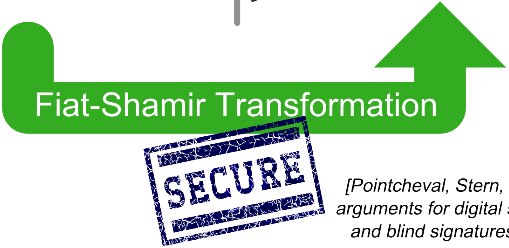
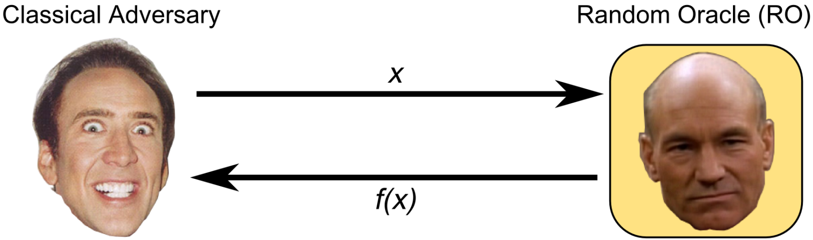


[Pointcheval, Stern, "Security arguments for digital signatures and blind signatures", 2000]
(ROM)

Classical VS Quantum



Classical Adversary

Random Oracle (RO)

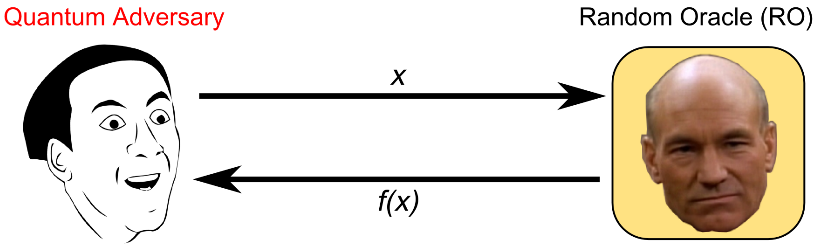
x

$f(x)$

Fiat-Shamir Transformation

SECURE

Classical VS Quantum



Quantum Adversary

Random Oracle (RO)

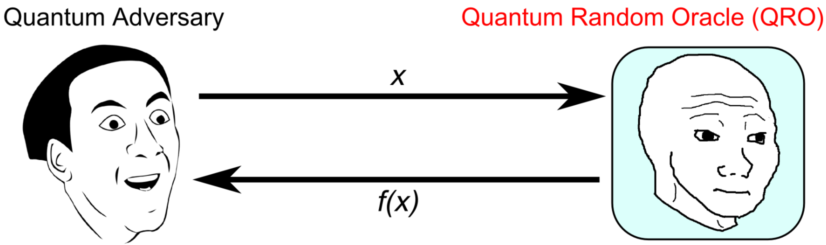
x

$f(x)$

Fiat-Shamir Transformation

SECURE

Classical VS Quantum



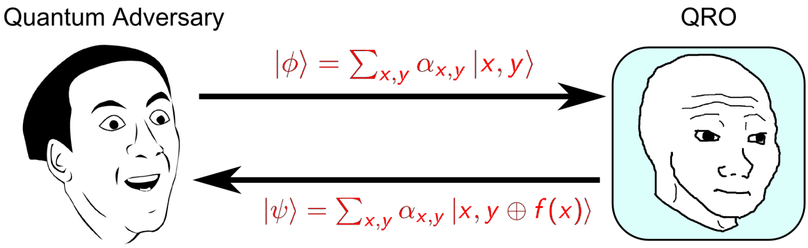
Quantum Adversary

Quantum Random Oracle (QRO)

Fiat-Shamir Transformation

SECURE

Classical VS Quantum



Quantum Adversary

QRO

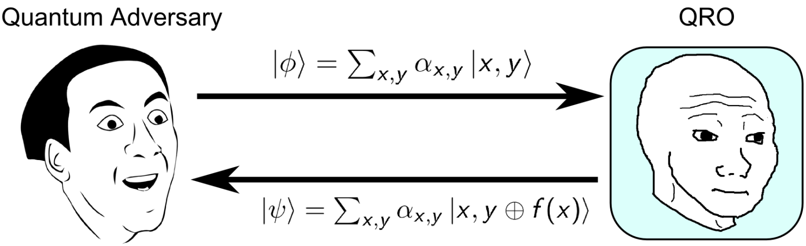
$|\phi\rangle = \sum_{x,y} \alpha_{x,y} |x, y\rangle$

$|\psi\rangle = \sum_{x,y} \alpha_{x,y} |x, y \oplus f(x)\rangle$

Fiat-Shamir Transformation

SECURE

Classical VS Quantum



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Fiat-Shamir Transformation

Security in the QROM

Problem: in the QROM* many of the techniques we use for security proofs do not usually work:

[Boneh,Dagdelen,Fischlin,Lehmann,Schaffner,Zhandry,'Random Oracles in a Quantum World'*,2010]

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All these things do not work!

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Security in the QROM

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Open question

Is the Fiat–Transformation secure in the QROM?

[Boneh,Dagdelen,Fischlin,Lehmann,Schaffner,Zhandry,'Random Oracles in a Quantum World'*,2010]

Outline of our work

Impossibility result

For certain schemes, we use a **meta-reduction** to rule out the existence of (a large class of) possible security proofs.

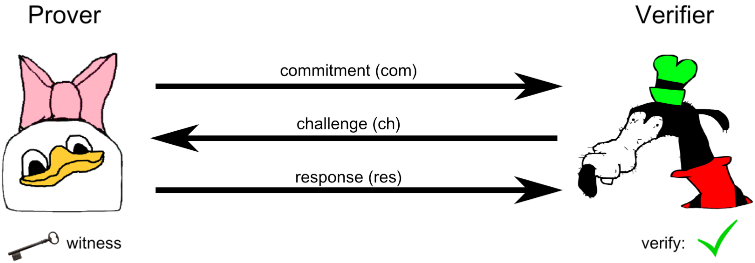
Positive result

For other schemes, we give a proof of security by defining and using **oblivious commitments**.

Secure instantiation

We provide a **generic patch** to harden existing schemes with a small overhead, and we give an **example instantiation** based on a recent lattice-based signature scheme.

Identification scheme



Prover

Verifier

commitment (com)

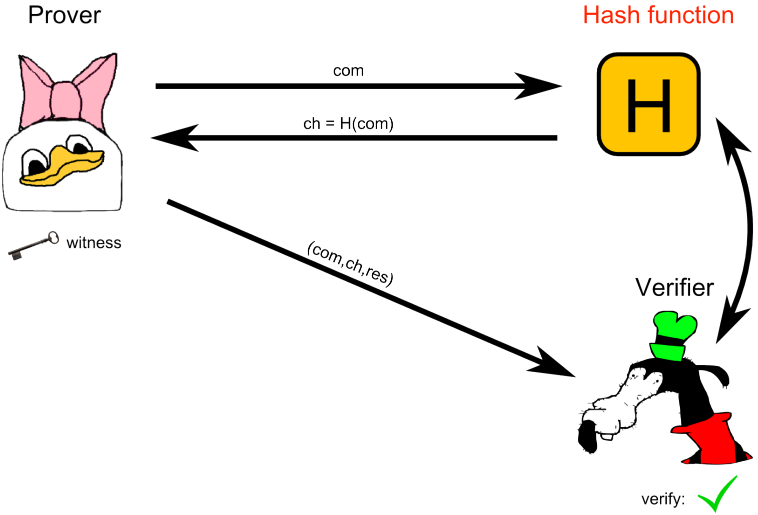
challenge (ch)

response (res)

witness

verify: ✓

Identification scheme



Prover

Hash function

com

ch = H(com)

H

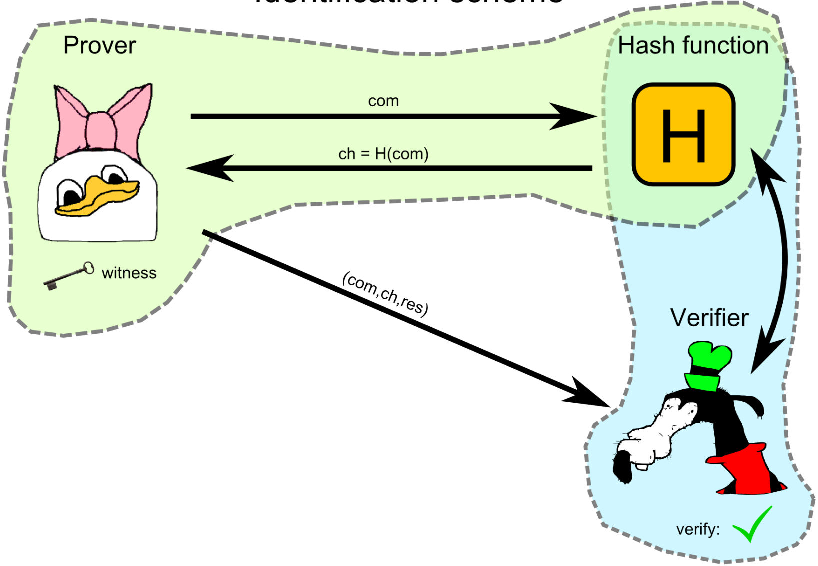
witness

(com,ch,res)

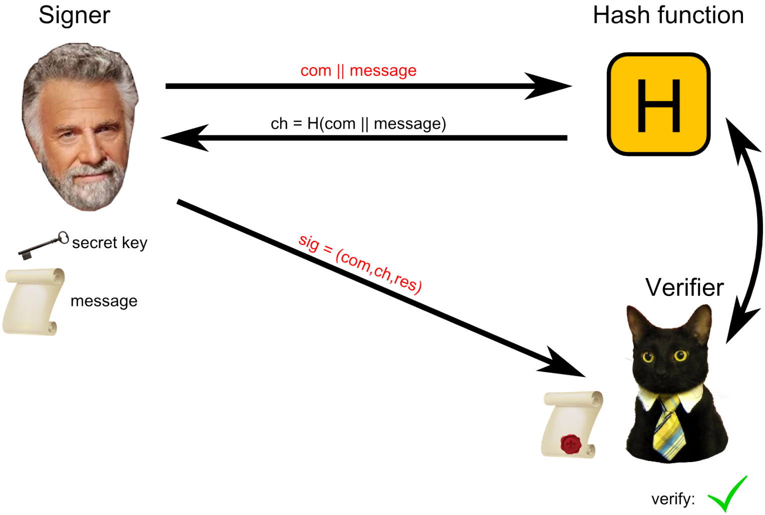
Verifier

verify: ✓

Identification scheme



Signature scheme



Signer

Hash function

com || message

ch = H(com || message)

H

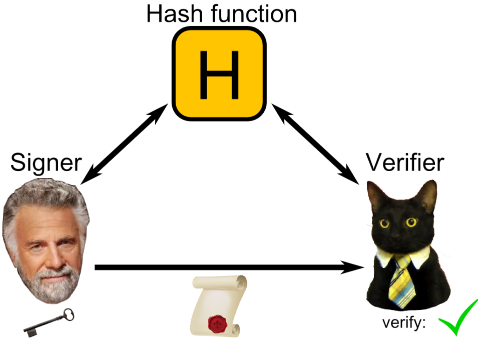
secret key

message

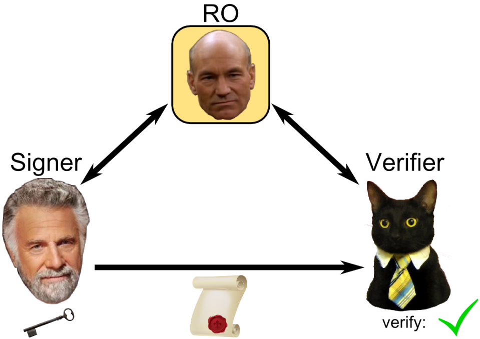
sig = (com,ch,res)

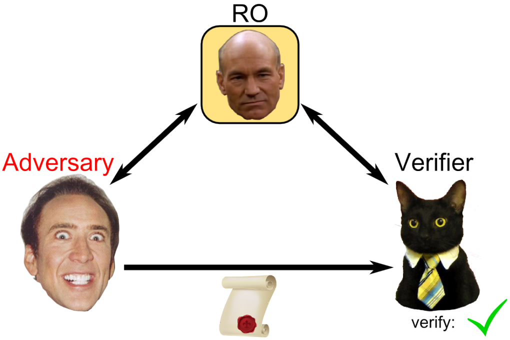
Verifier

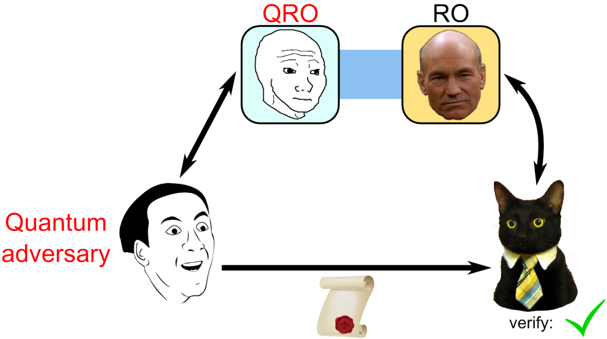
verify: ✓



Hash function
H
Signer
Verifier
verify: ✓


RO
Signer
Verifier
verify: ✓


RO
Adversary
Verifier
verify: ✓

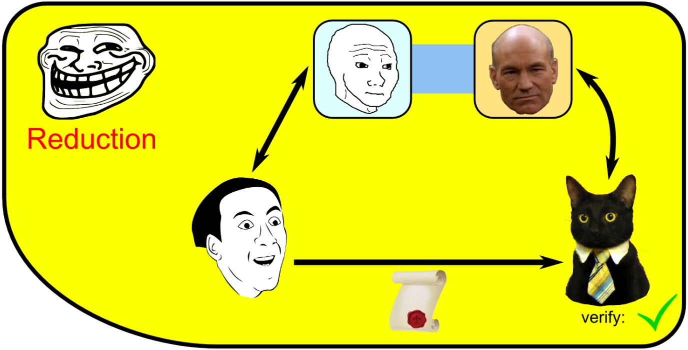


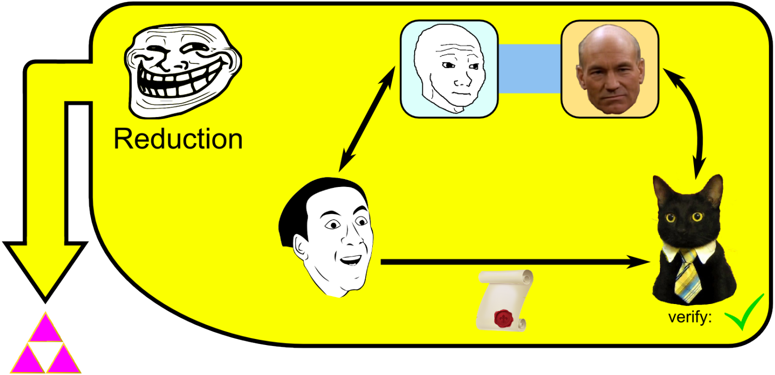
QRO

RO

Quantum adversary

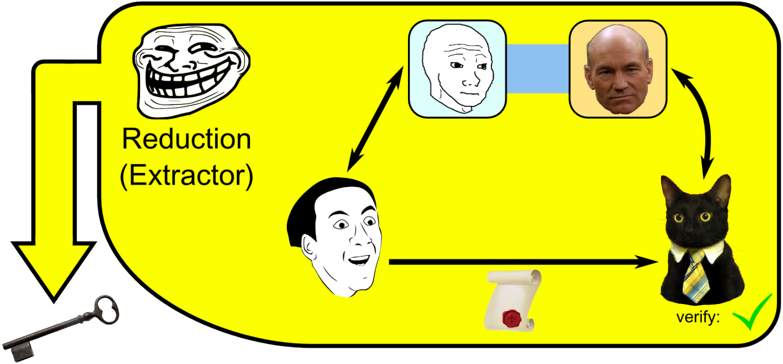
verify: ✓

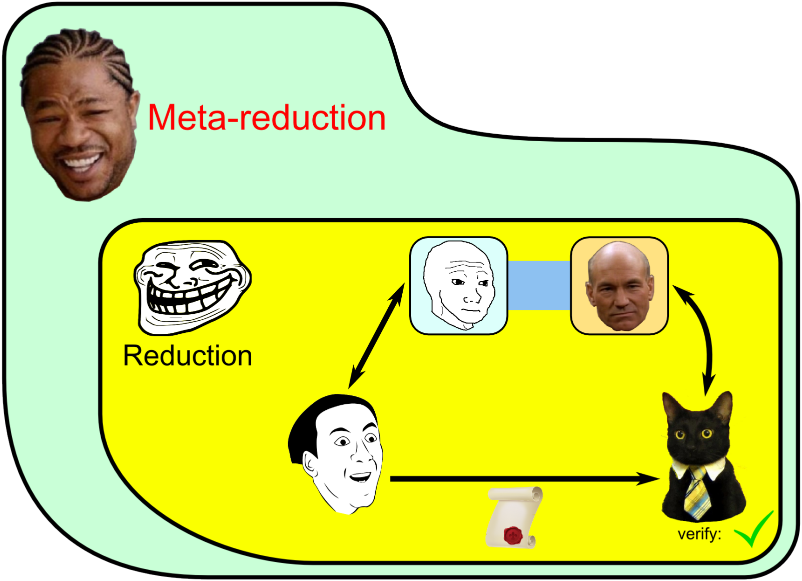

Reduction
verify: ✓



Reduction

verify: ✓

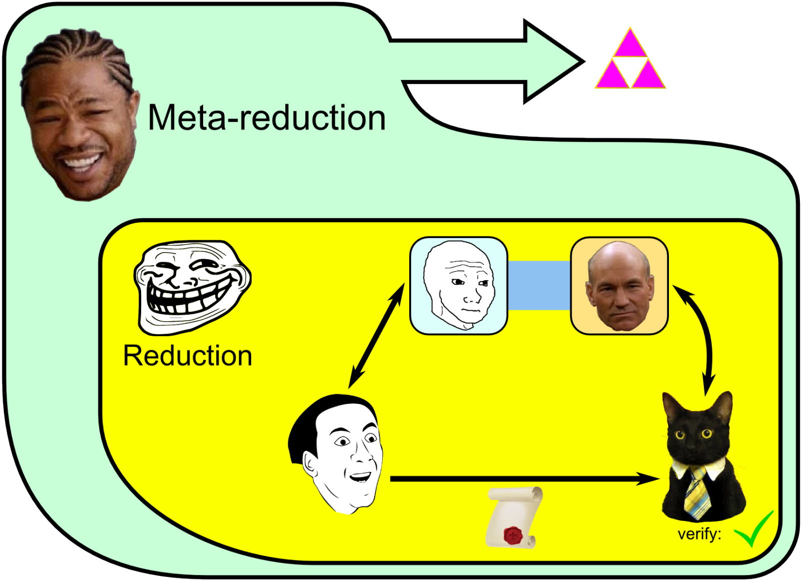

Reduction
(Extractor)
verify: ✓

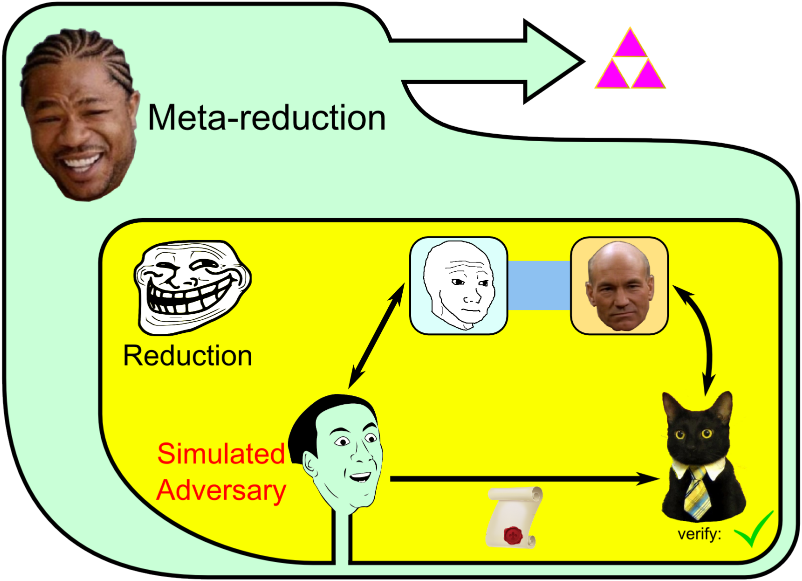


Meta-reduction

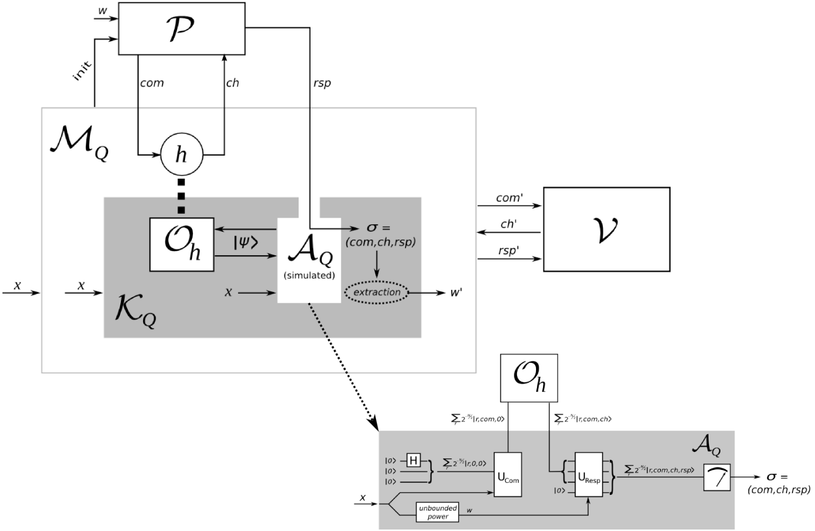
Reduction

verify: ✓


Meta-reduction
Reduction
verify: ✓


Meta-reduction
Reduction
Simulated Adversary
verify: ✓

Our Meta-Reduction



Impossibility result

Theorem:

No Fiat–Shamir signature scheme admits efficient black-box extractors, provided underlying identification scheme has:

- **witness-independent commitments**
- **active security**

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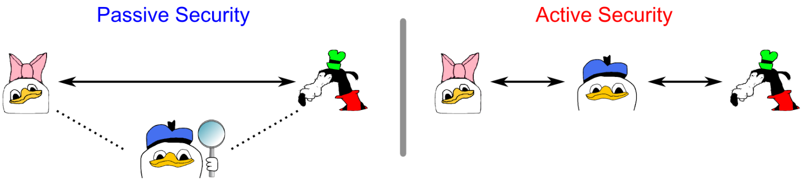

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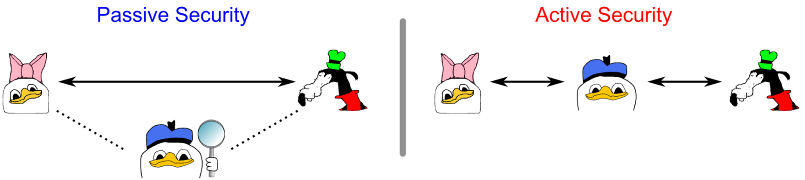

Notice: **passive security** is enough to obtain secure signature schemes via the Fiat–Shamir transform.

Positive result

Idea

Remove active security from underlying identification scheme

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Identification schemese with **Oblivious Commitments**

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with normal commitment:

w,x

$ → s

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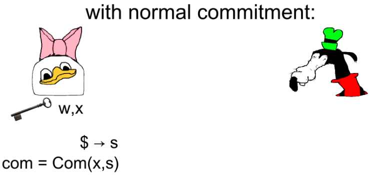


with normal commitment:

w,x

$ → s

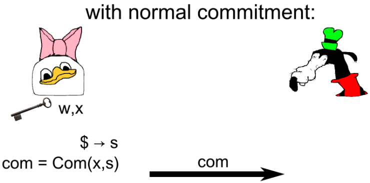
com = Com(x,s)

Positive result

Idea

Remove active security from underlying identification scheme

Identification schemese with **Oblivious Commitments**



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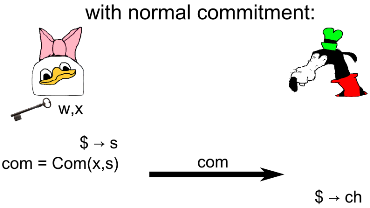

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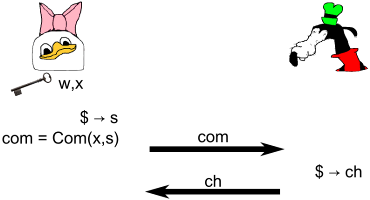

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← ch

$ → ch

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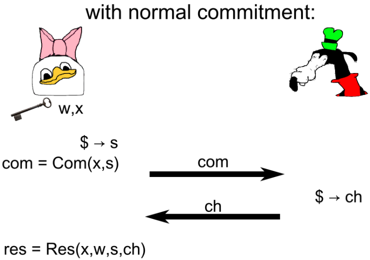

with normal commitment:

w,x

$ → s

com = Com(x,s)

com →

$ → ch

← ch

res = Res(x,w,s,ch)

Positive result

Idea

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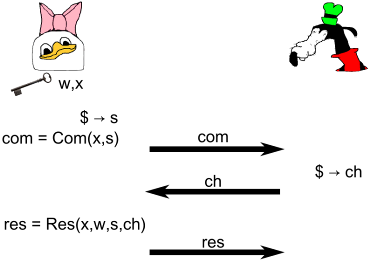
with normal commitment:



w,x

$\$ \to s$

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$\$ \to ch$

ch

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res

Positive result

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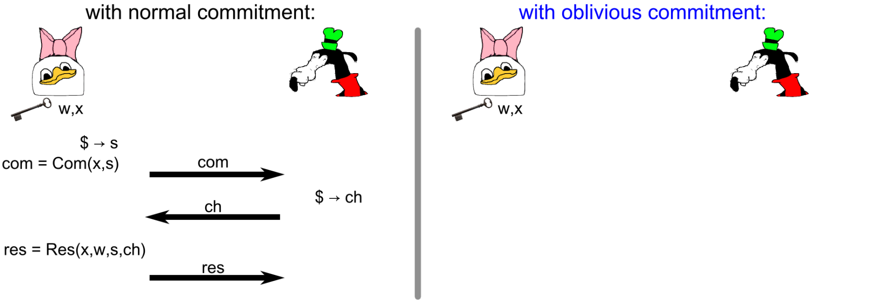

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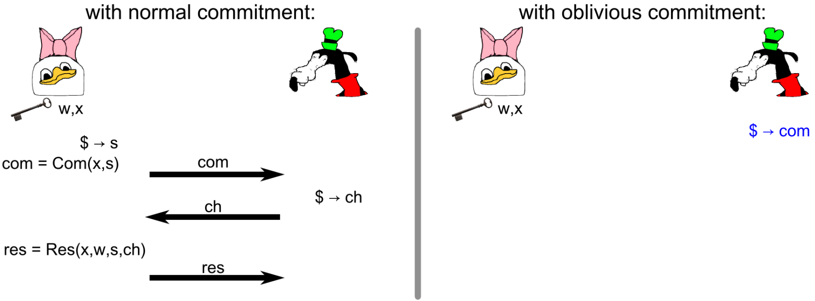

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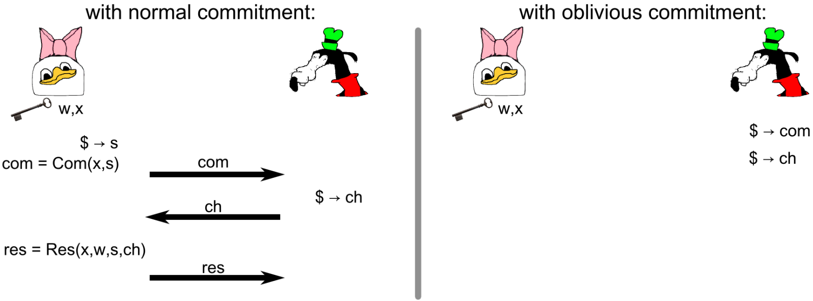

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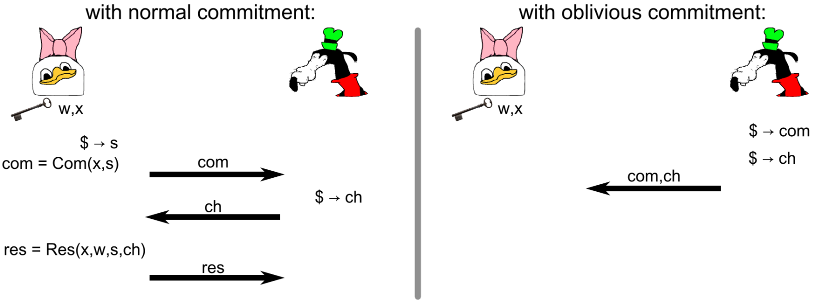

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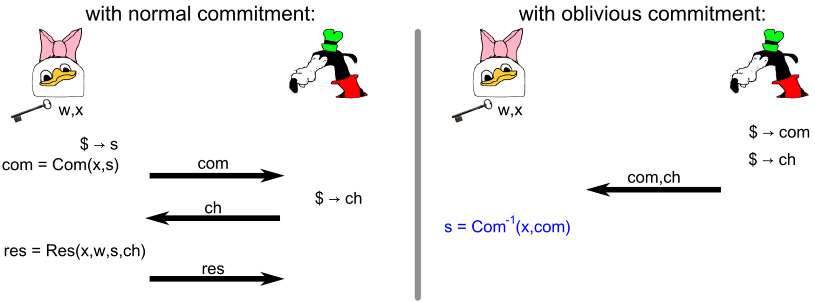

with normal commitment:

w,x

$\$ \rightarrow s$

com = Com(x,s)

com

ch

$\$ \rightarrow ch$

res = Res(x,w,s,ch)

res

with oblivious commitment:

w,x

$\$ \rightarrow com$

$\$ \rightarrow ch$

com,ch

$s = Com^{-1}(x,com)$

Positive result

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Remove active security from underlying identification scheme

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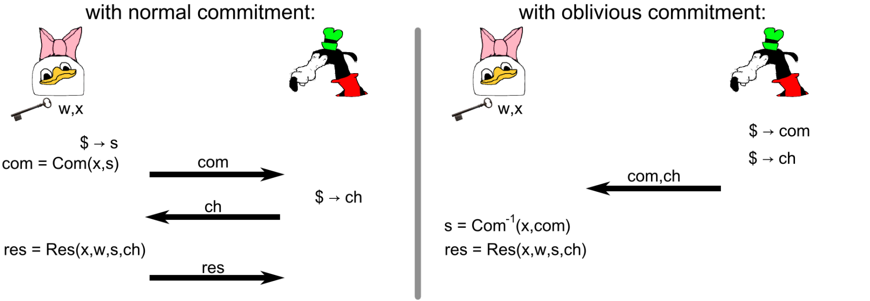

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ch

$ → ch

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w,x

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com,ch

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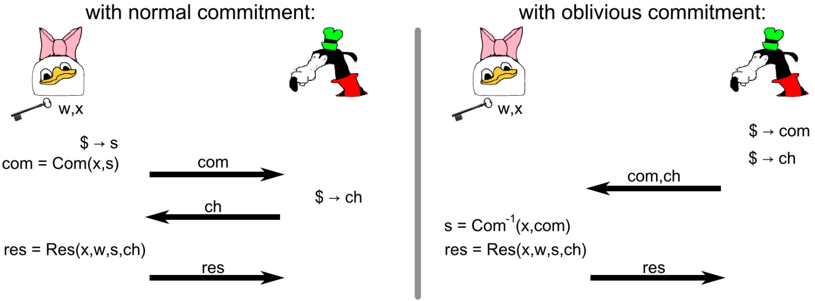

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w,x

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com

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ch

res = Res(x,w,s,ch)

res

with oblivious commitment:

w,x

$ → com

$ → ch

com,ch

s = Com^{-1}(x,com)

res = Res(x,w,s,ch)

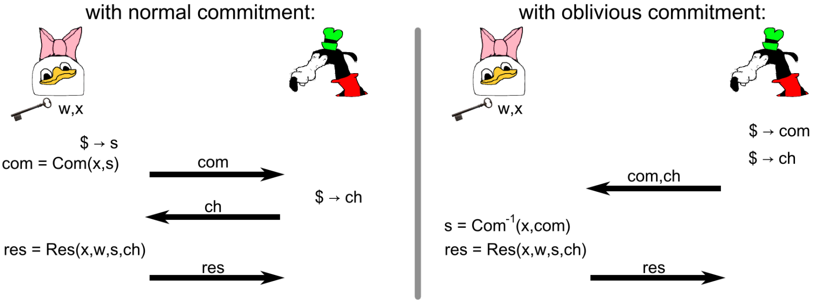
res

Positive result

Idea

Remove active security from underlying identification scheme

Identification schemese with **Oblivious Commitments**



with normal commitment:

w,x

$ → s

com = Com(x,s)

com

$ → ch

ch

res = Res(x,w,s,ch)

res

with oblivious commitment:

w,x

$ → com

$ → ch

com,ch

s = Com^{-1}(x,com)

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res

Oblivious commitments remove active security!

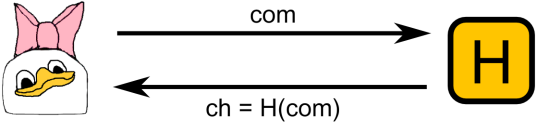
Positive result

How to apply Fiat–Shamir with oblivious commitment schemes?

Positive result

How to apply Fiat–Shamir with oblivious commitment schemes?

Our patch:



Positive result

How to apply Fiat–Shamir with oblivious commitment schemes?

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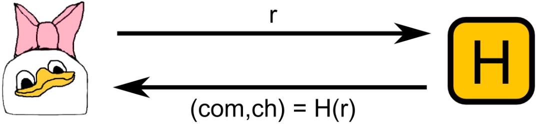

Positive result

How to apply Fiat–Shamir with oblivious commitment schemes?

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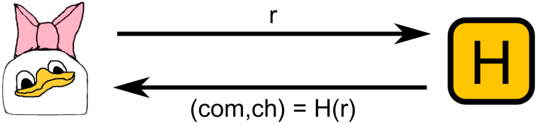
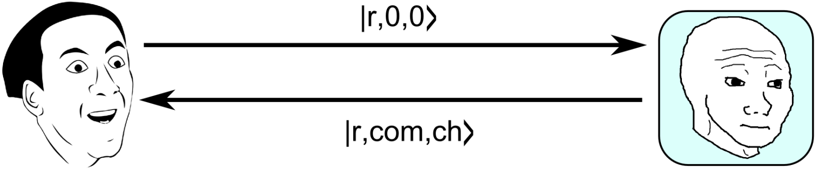


Theorem:

The Fiat–Shamir transformation of an oblivious commitment identification scheme yields an existentially unforgeable secure signature scheme in the QROM.

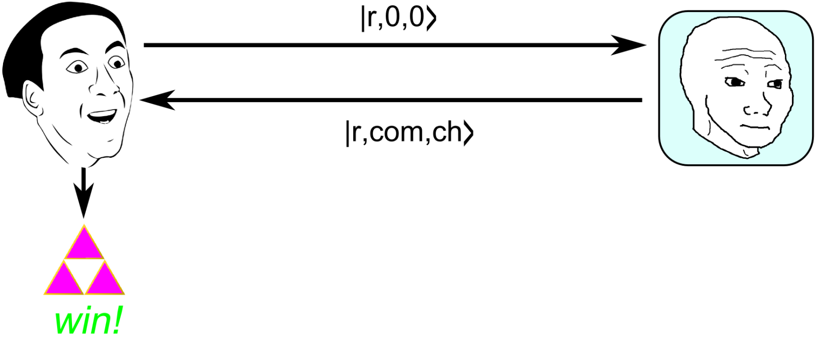
How our positive result works



$|r,0,0\rangle$

$|r,\mathrm{com},\mathrm{ch}\rangle$

How our positive result works



$|r,0,0\rangle$

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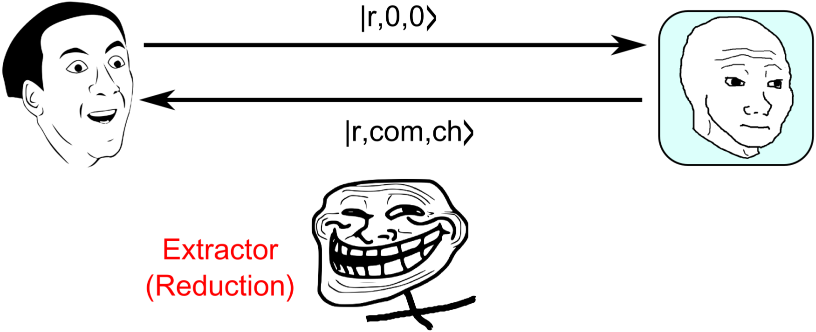
win!

How our positive result works

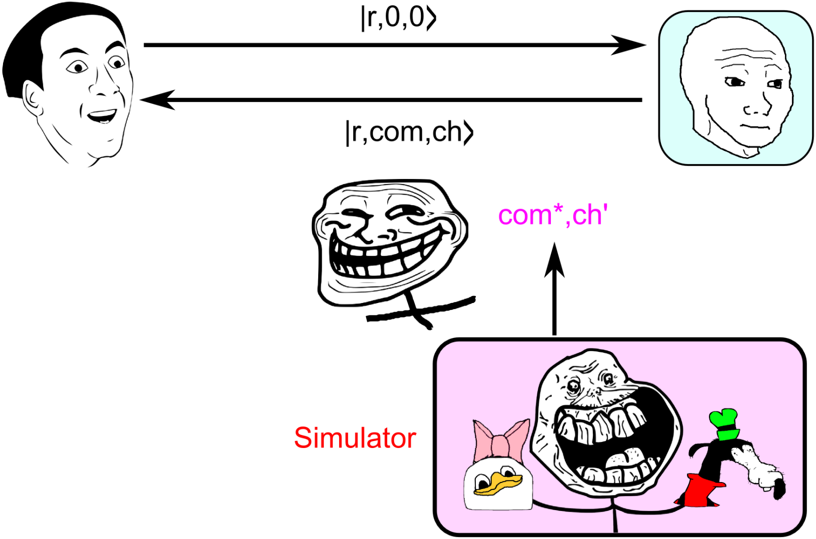


$|r,0,0\rangle$

$|r,com,ch\rangle$

Extractor
(Reduction)

How our positive result works



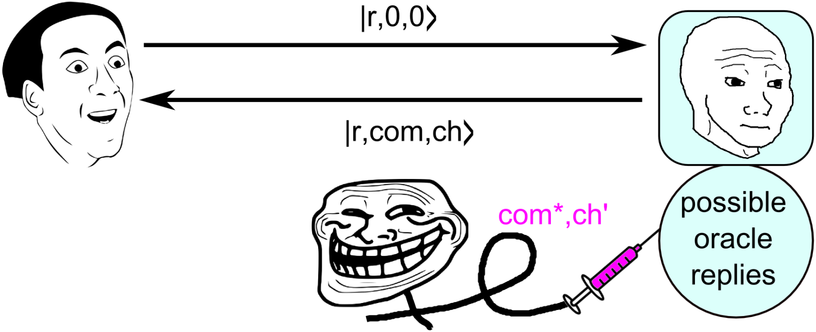
$|r,0,0\rangle$

$|r,com,ch\rangle$

com*,ch'

Simulator

How our positive result works



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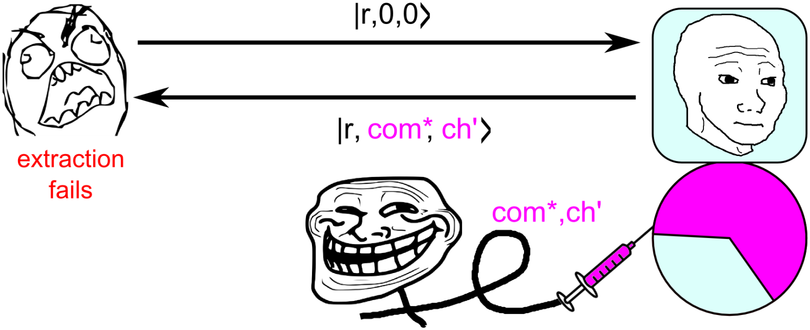

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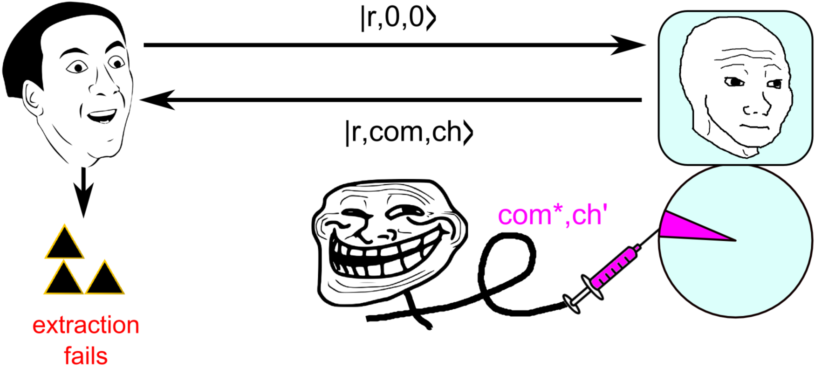

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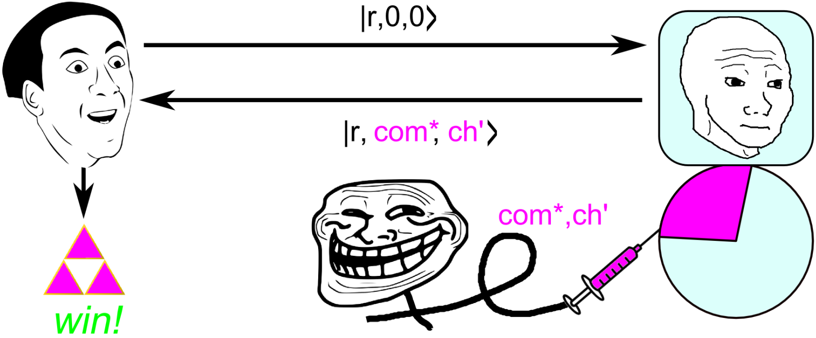

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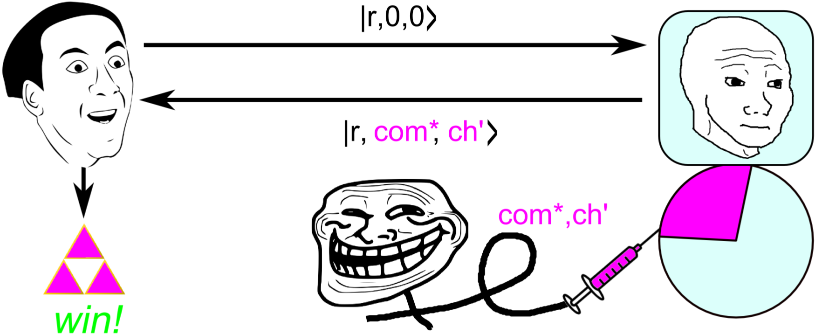


$|r,0,0\rangle$

$|r, \text{com*}, \text{ch'}\rangle$

com*,ch'

win!

$$\Pr\left[\mathit{success}\right] \geq \delta\epsilon - \frac{8}{3}q_H^4\delta^2 - q_S \cdot 4\sqrt{(q_H + q_S)\cdot 2^{-\frac{n}{2}}}$$
$$- \left(q_S q_H + \frac{(q_S - 1)^2}{2}\right)\cdot 2^{-n} - negl(\epsilon)$$

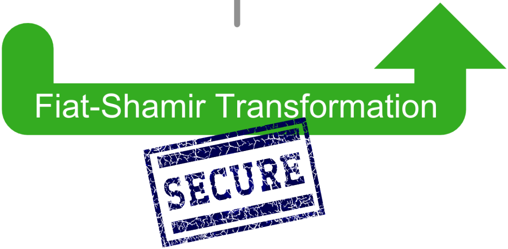
A surprising result

Weaker
Identification scheme

(no active security)

Stronger
Signature scheme

(provably secure
in the QROM)



Fiat-Shamir Transformation

SECURE

An example instantiation

1. Start with a witness-independent, oblivious commitment identification scheme based on post-quantum primitives

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   Our choice: [Lyu12]

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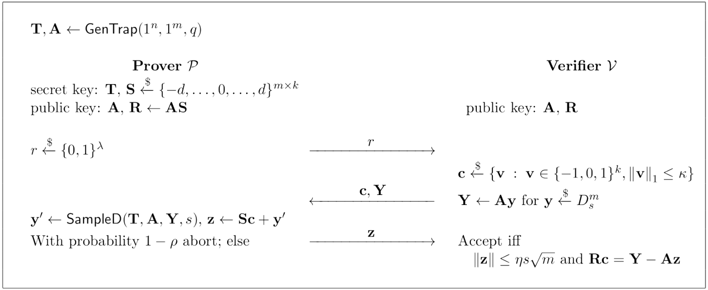
3. Let the verifier choose and send both *com* and *ch*
4. Prover uses a trapdoor to find preimage for the obtained oblivious commitment and completes protocol
5. Apply our 'patched' Fiat–Shamir transformation to resulting scheme.

[Lyu12]: V. Lyubashevsky, *'Lattice signatures without trapdoors'*, 2012

Our patched version of the Lyubashevsky scheme



$\mathbf{T}, \mathbf{A} \leftarrow \mathsf{GenTrap}(1^n, 1^m, q)$

Prover $\mathcal{P}$		**Verifier** $\mathcal{V}$
secret key: $\mathbf{T}$, $\mathbf{S} \overset{\$}{\leftarrow} \{-d, \ldots, 0, \ldots, d\}^{m \times k}$		
public key: $\mathbf{A}$, $\mathbf{R} \leftarrow \mathbf{AS}$		public key: $\mathbf{A}$, $\mathbf{R}$
$r \overset{\$}{\leftarrow} \{0,1\}^\lambda$	$\xrightarrow{r}$	
		$\mathbf{c} \overset{\$}{\leftarrow} \{\mathbf{v} \ : \ \mathbf{v} \in \{-1,0,1\}^k, \Vert\mathbf{v}\Vert_1 \leq \kappa\}$
	$\xleftarrow{\mathbf{c}, \mathbf{Y}}$	$\mathbf{Y} \leftarrow \mathbf{Ay}$ for $\mathbf{y} \overset{\$}{\leftarrow} D_s^m$
$\mathbf{y}' \leftarrow \mathsf{SampleD}(\mathbf{T}, \mathbf{A}, \mathbf{Y}, s)$, $\mathbf{z} \leftarrow \mathbf{Sc} + \mathbf{y}'$		
With probability $1 - \rho$ abort; else	$\xrightarrow{\mathbf{z}}$	Accept iff
		$\Vert\mathbf{z}\Vert \leq \eta s \sqrt{m}$ and $\mathbf{Rc} = \mathbf{Y} - \mathbf{Az}$

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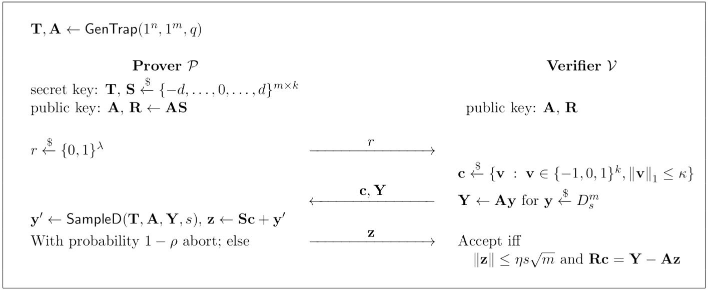

$\mathbf{T}, \mathbf{A} \leftarrow \mathsf{GenTrap}(1^n, 1^m, q)$

Prover $\mathcal{P}$

secret key: $\mathbf{T}$, $\mathbf{S} \xleftarrow{\$} \{-d, \ldots, 0, \ldots, d\}^{m \times k}$

public key: $\mathbf{A}$, $\mathbf{R} \leftarrow \mathbf{AS}$

Verifier $\mathcal{V}$

public key: $\mathbf{A}$, $\mathbf{R}$

$r \xleftarrow{\$} \{0,1\}^\lambda$ $\xrightarrow{\quad r \quad}$

$\mathbf{c} \xleftarrow{\$} \{\mathbf{v} \ : \ \mathbf{v} \in \{-1,0,1\}^k, \|\mathbf{v}\|_1 \leq \kappa\}$

$\xleftarrow{\quad \mathbf{c}, \mathbf{Y} \quad}$ $\mathbf{Y} \leftarrow \mathbf{Ay}$ for $\mathbf{y} \xleftarrow{\$} D_s^m$

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Accept iff

$\|\mathbf{z}\| \leq \eta s \sqrt{m}$ and $\mathbf{Rc} = \mathbf{Y} - \mathbf{Az}$

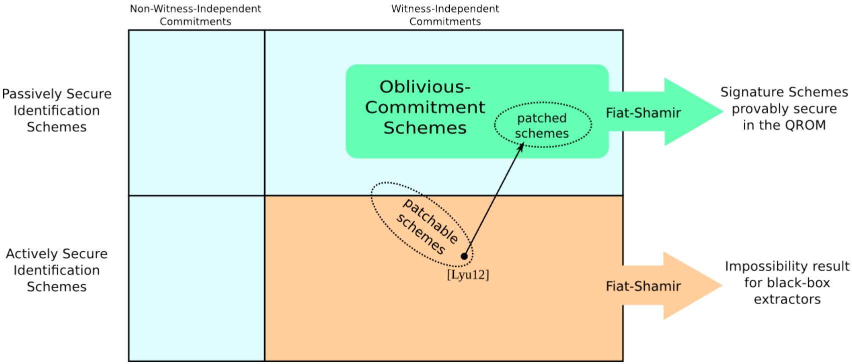
Similar to [GPV08] with hash-and-sign, also proven secure in [BZ13]

[GPV08]:Gentry,Peikert,Vaikuntanathan,*'Trapdoors for hard lattices and new cryptographic constructions'*,2008

[BZ13]:Boneh,Zhandry,*'Secure signatures and chosen ciphertext security in a post-quantum world'*,2013

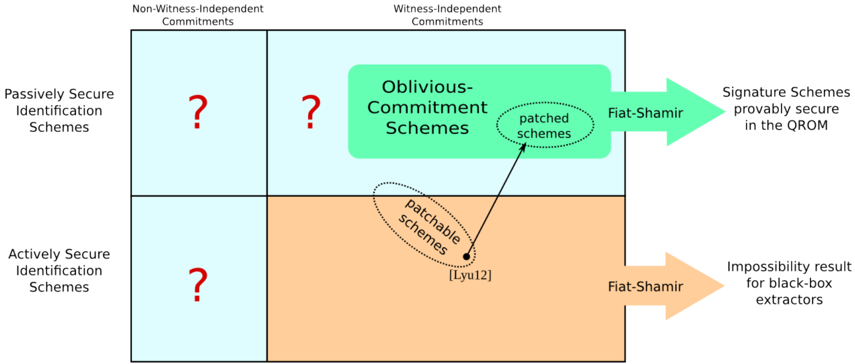
Conclusions

The Fiat–Shamir Transformation in the QROM


Non-Witness-Independent Commitments
Witness-Independent Commitments
Passively Secure Identification Schemes
Actively Secure Identification Schemes
Oblivious-Commitment Schemes
patched schemes
patchable schemes
[Lyu12]
Fiat-Shamir
Signature Schemes provably secure in the QROM
Fiat-Shamir
Impossibility result for black-box extractors

Conclusions

The Fiat–Shamir Transformation in the QROM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Open questions

End of this talk

Thanks for your attention!

tommaso@gagliardoni.net

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EC SPRIDE
EUROPEAN CENTER FOR
SECURITY AND PRIVACY BY DESIGN