Semantic Security and Indistinguishability in the Quantum World

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CASED and EC-SPRIDE and TU Darmstadt

May 6th, 2015
Lugano, Switzerland

[1] from a joint work with Andreas Hülsing and Christian Schaffner

Introduction

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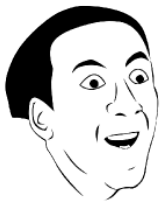
Starring



The Adversary

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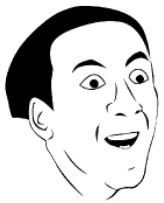

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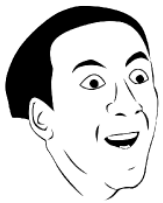

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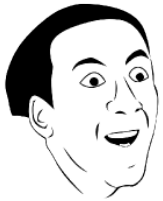

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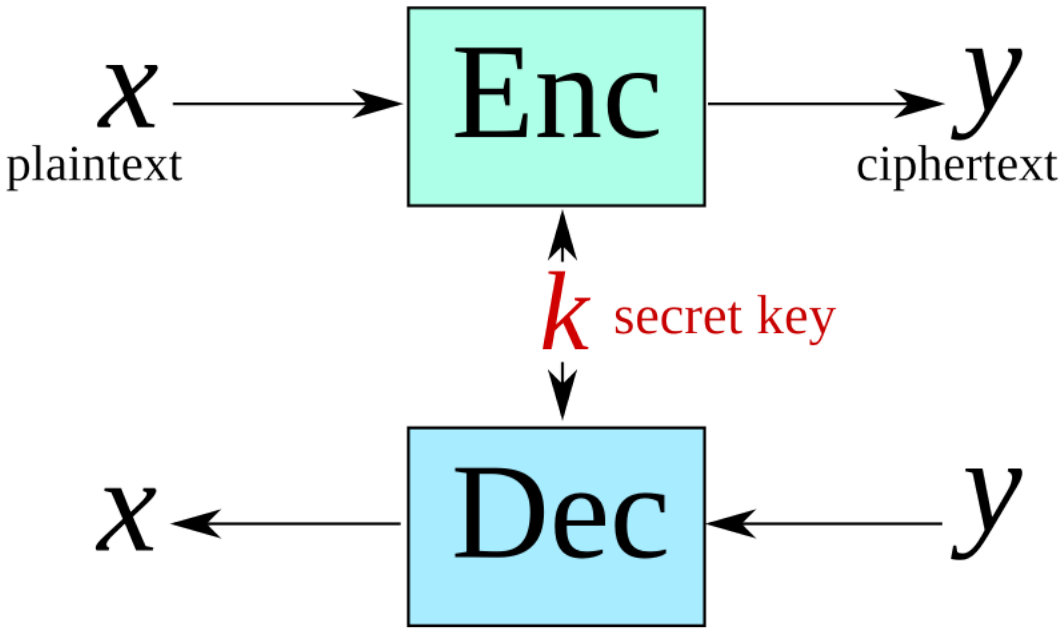
The Quantum Power

Introduction

Let's focus on symmetric-key encryption schemes



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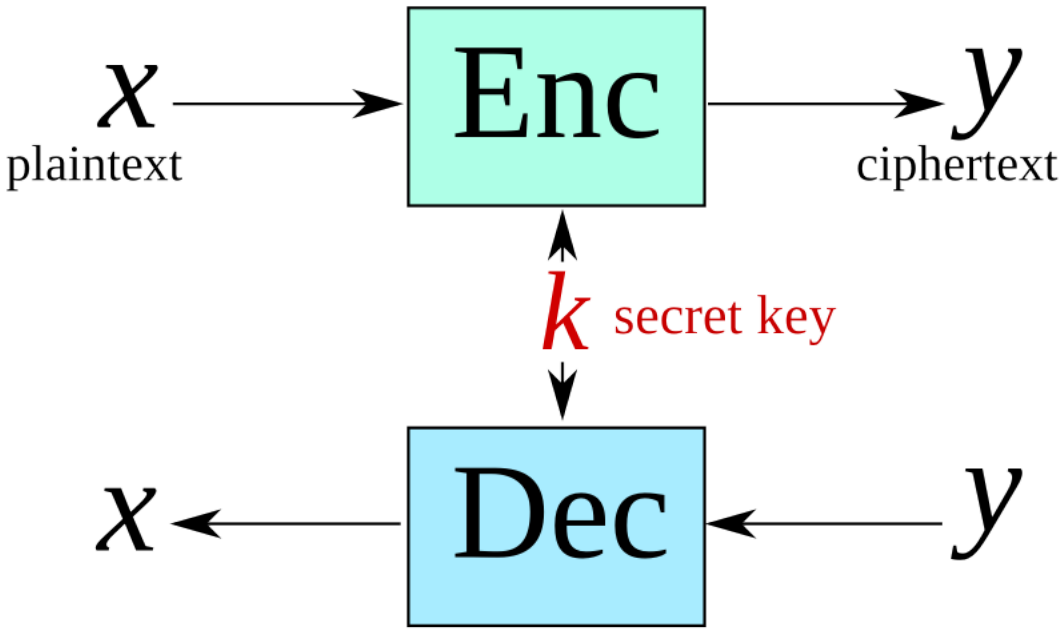

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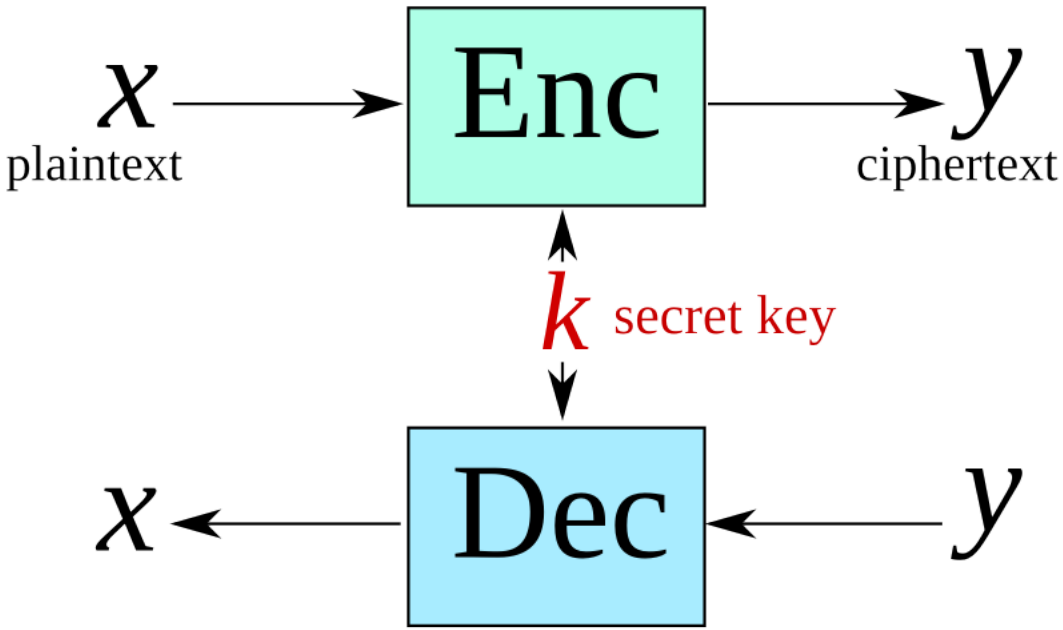

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'5dk8k4s0lQ1t6ss3hz01' $\Rightarrow$ Length$(x) = 20$ Bytes

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Game-based security: $\mathcal{A}$ and $\mathcal{S}$ play an interactive game in two different 'worlds' against $\mathcal{C}$.

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Classical Semantic Security (SEM)

For any efficient adversary $\mathcal{A}$ there exists an efficient simulator $\mathcal{S}$ such that:

$$|\Pr[\mathcal{A}(\mathsf{Enc}_k(x), h(x)) = f(x)] - \Pr[\mathcal{S}(h(x)) = f(x)]| \leq \mathsf{negl}(n).$$

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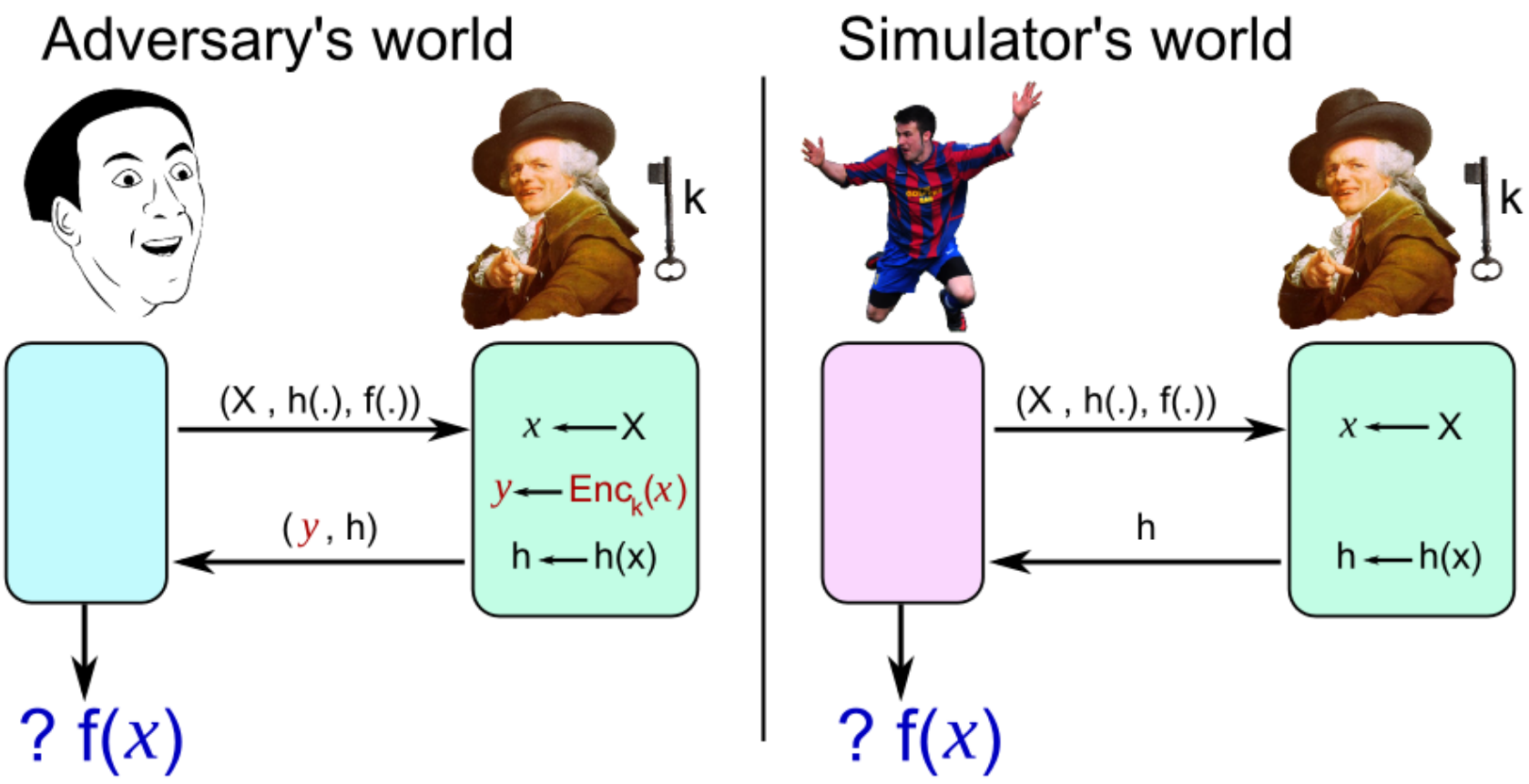

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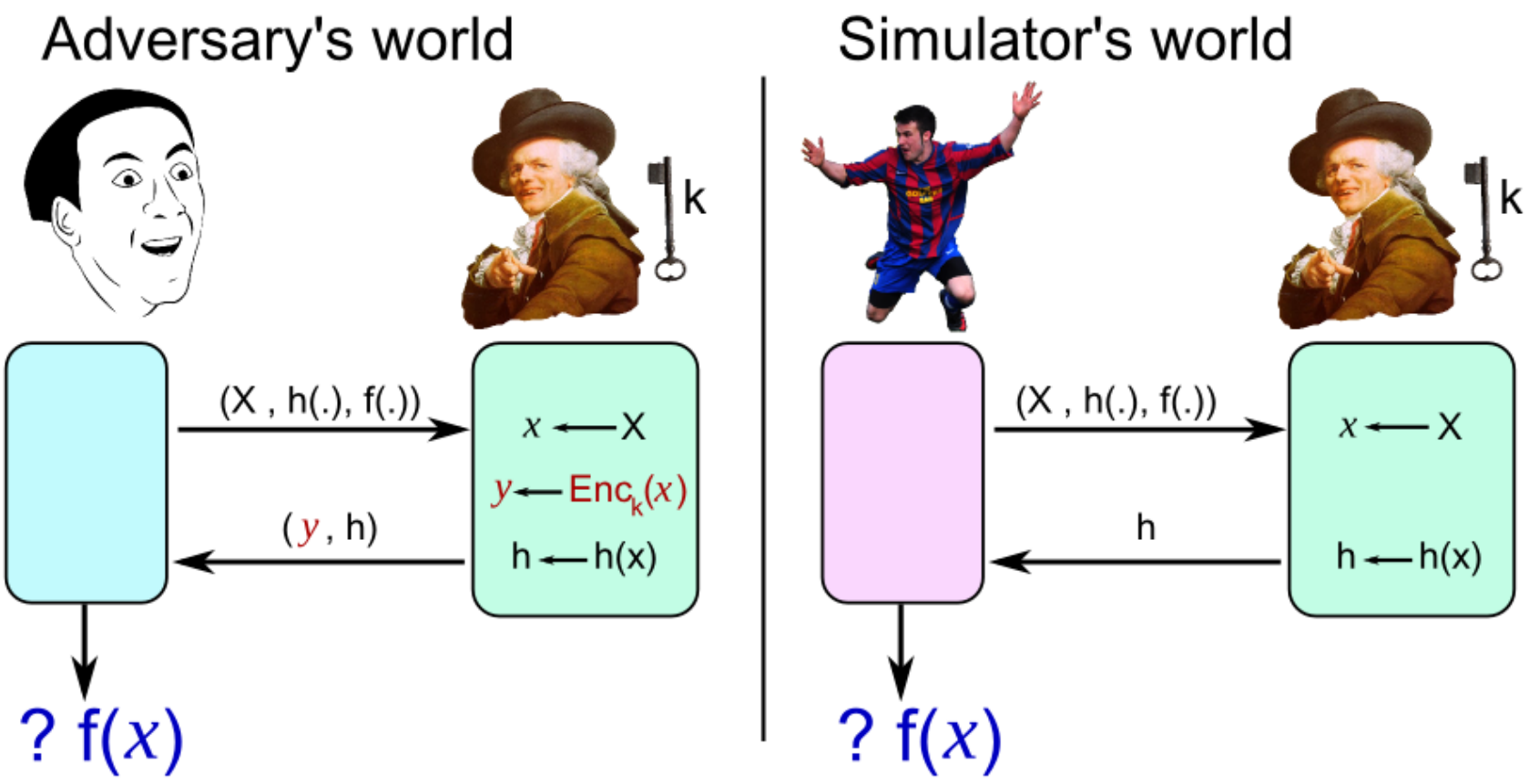

This definition is cumbersome.

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IND challenge query: $\mathcal{A}$ sends $\mathcal{C}$ two plaintexts $x_0, x_1 \in \mathcal{M}$.

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Classical Indistinguishability (IND)

For any efficient adversary $\mathcal{A}$ and any $x_0, x_1 \in \mathcal{M}$:

$$\left|\Pr[\mathcal{A}(y) = b] - \frac{1}{2}\right| \leq \mathsf{negl}(n).$$

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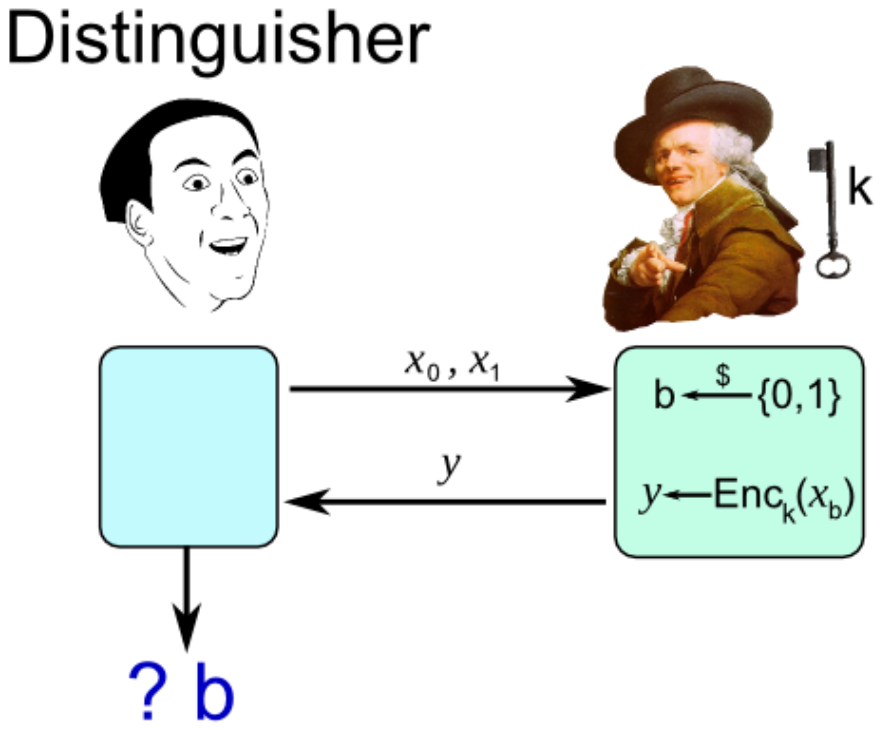

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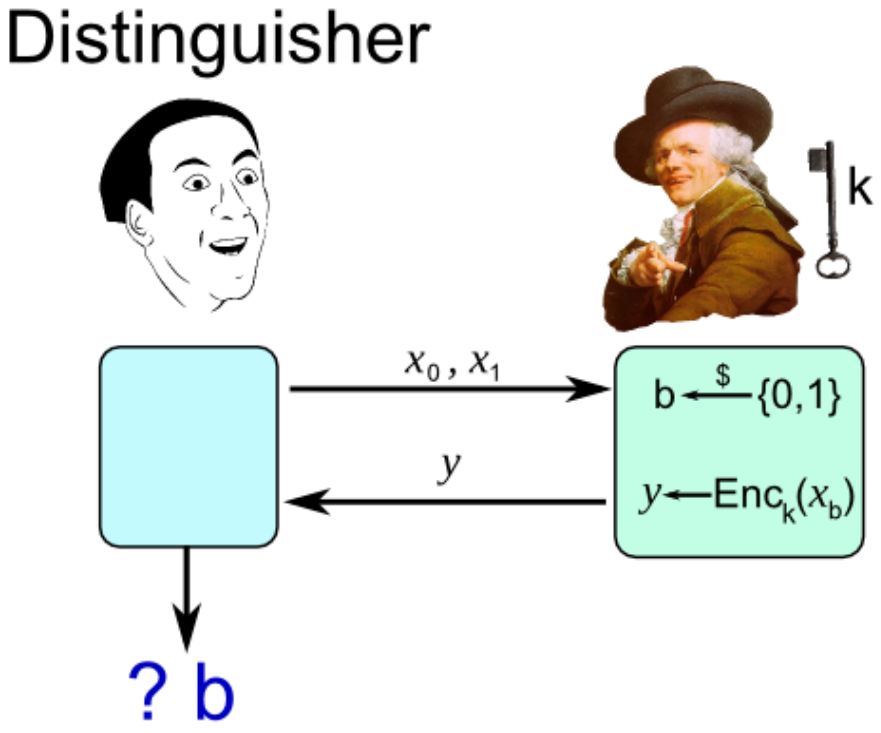

Theorem

IND $\iff$ SEM.

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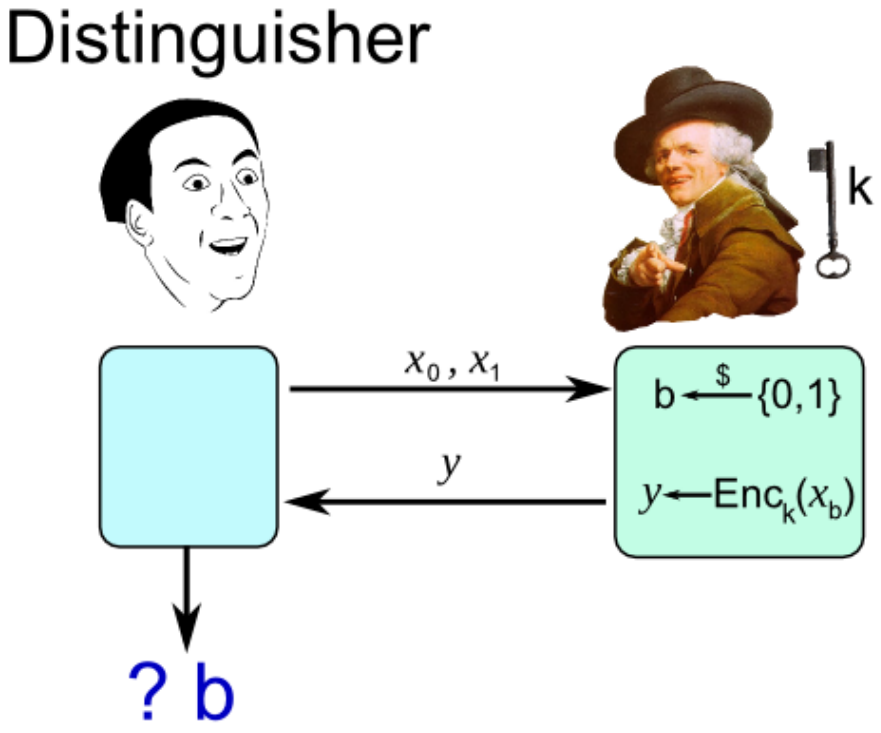

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(many other equivalent formulations of IND and SEM)

Chosen Plaintext Attack

CPA 'learning' phase: $\mathcal{A}$ sends $\mathcal{C}$ up to $q = \text{poly}(n)$ plaintexts $x_1, \ldots, x_q \in \mathcal{M}$ (possibly adaptively).

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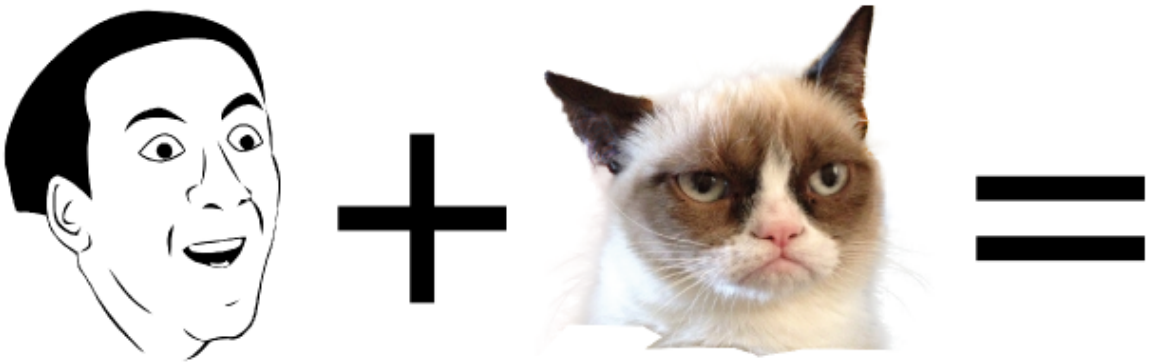
IND-CPA $\iff$ SEM-CPA.

Note: deterministic schemes are insecure $\Rightarrow$ need for randomization.

Quantum Adversaries



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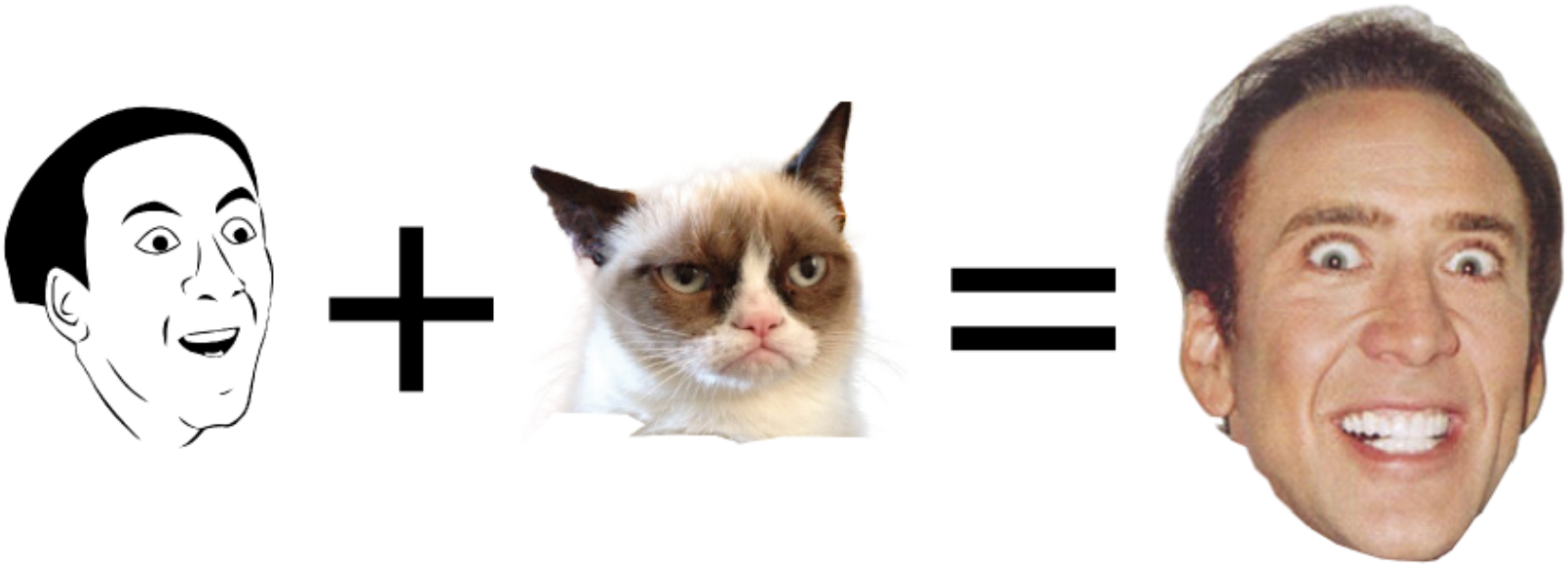

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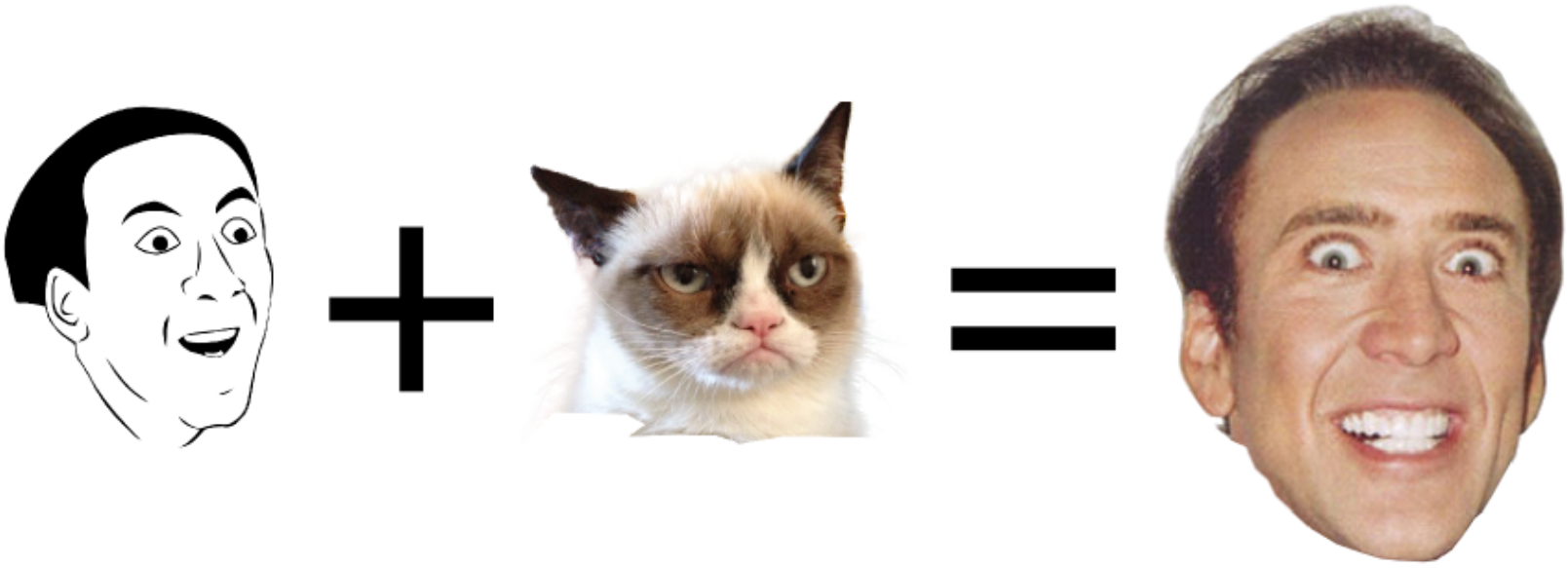

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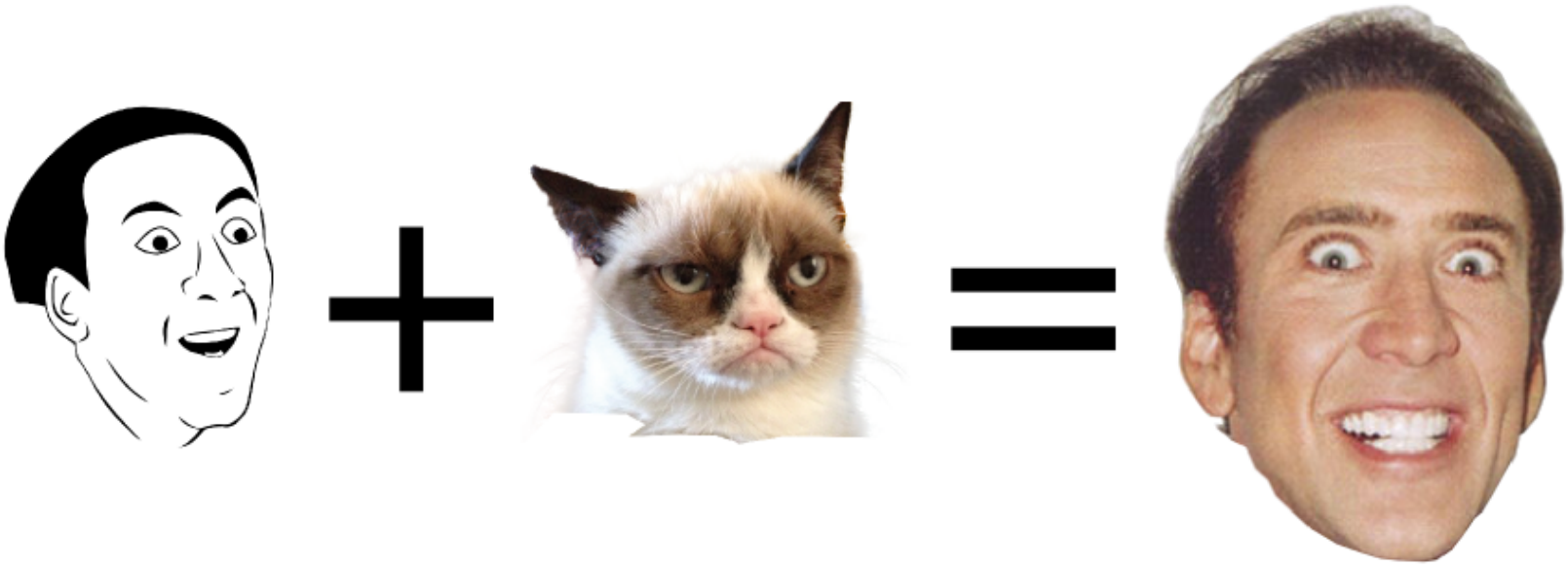

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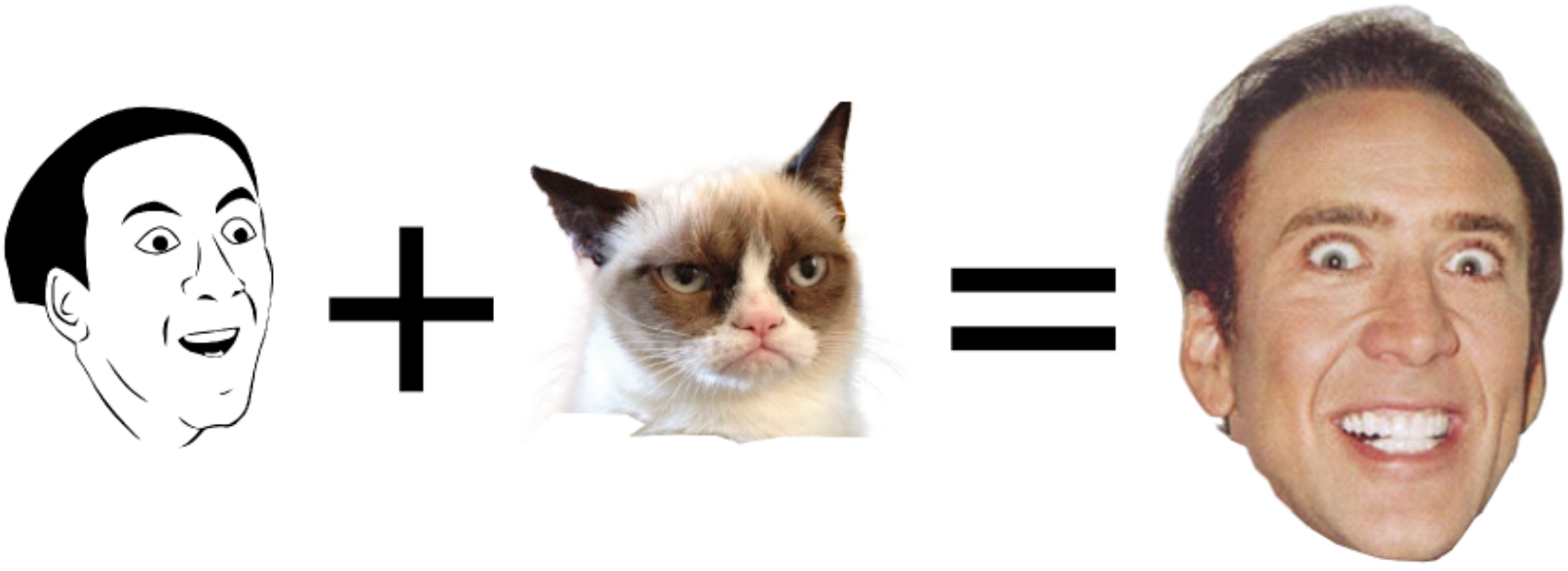

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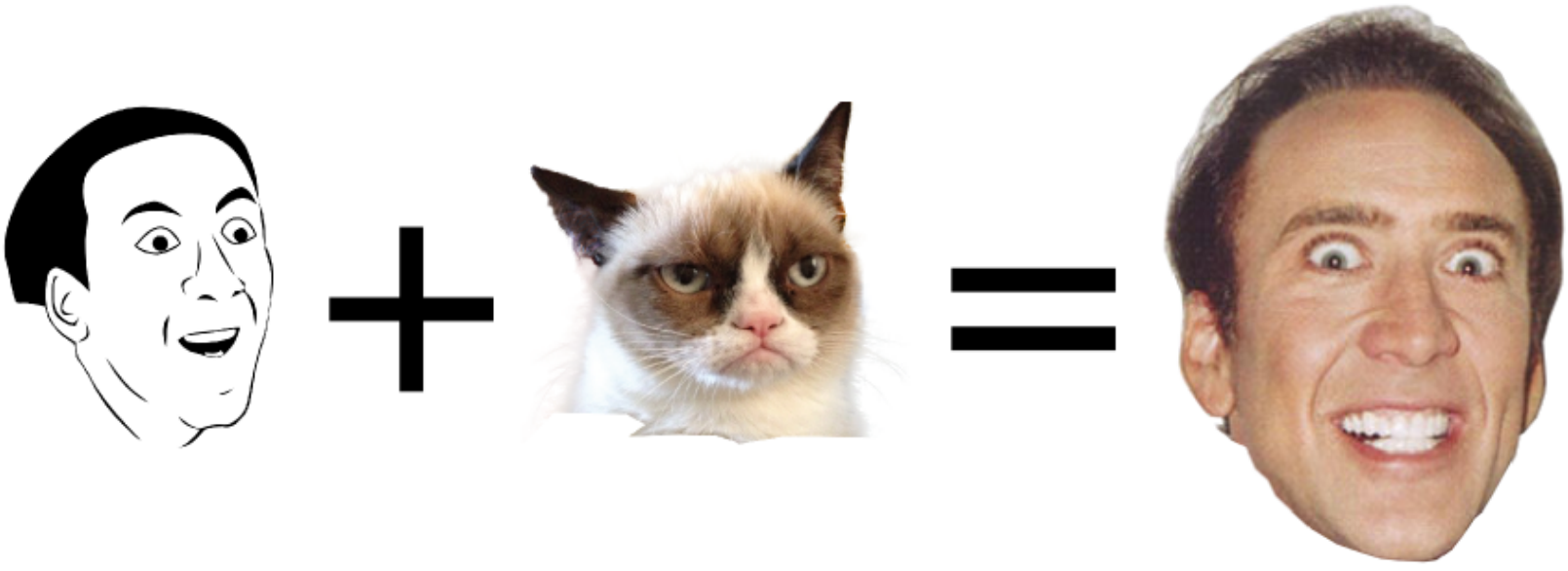

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This is not enough!!!

Quantum Access

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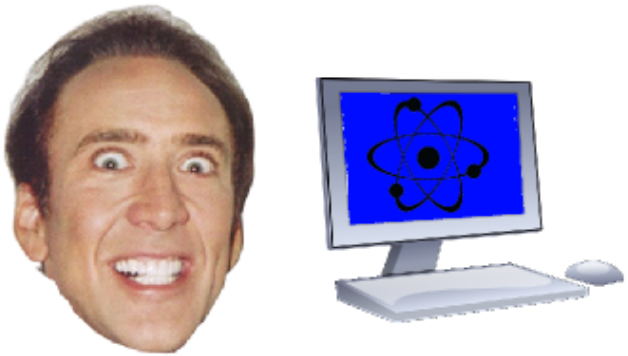

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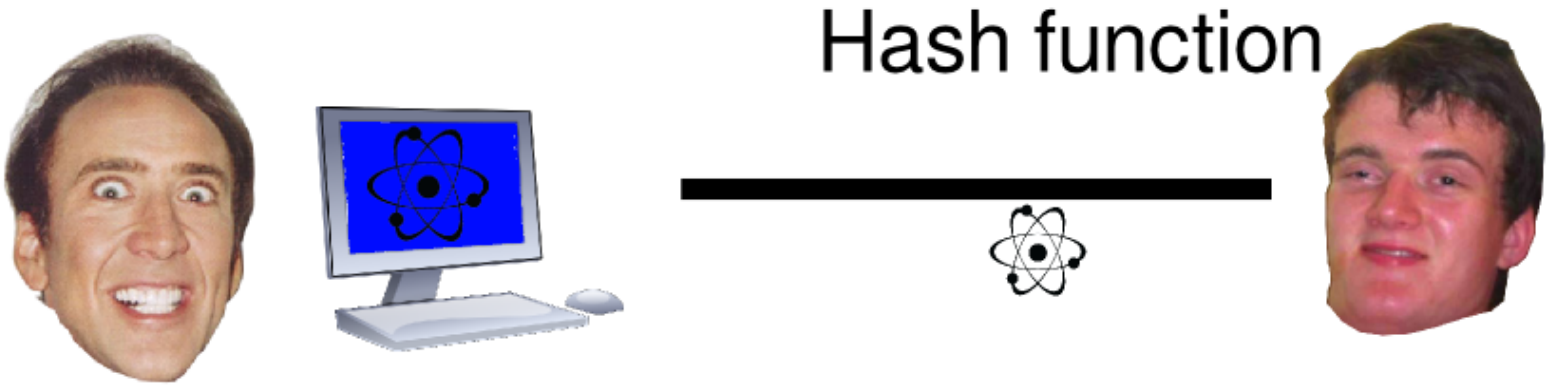

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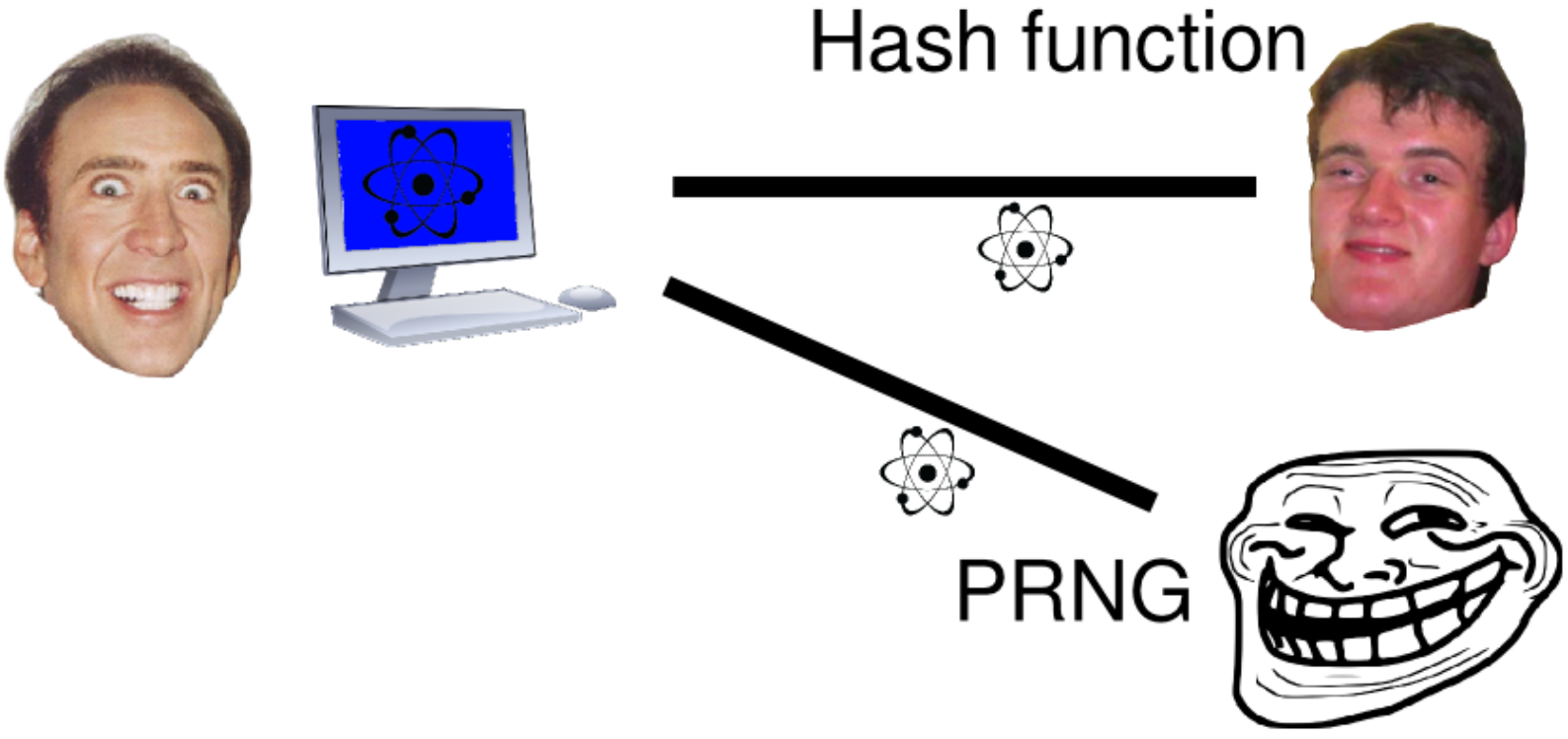

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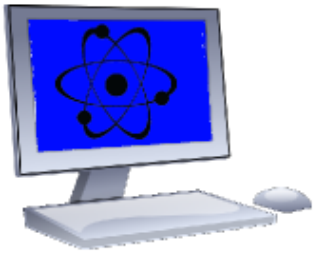

Hash function

PRNG

Encryption Oracle

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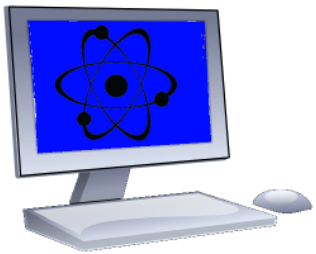

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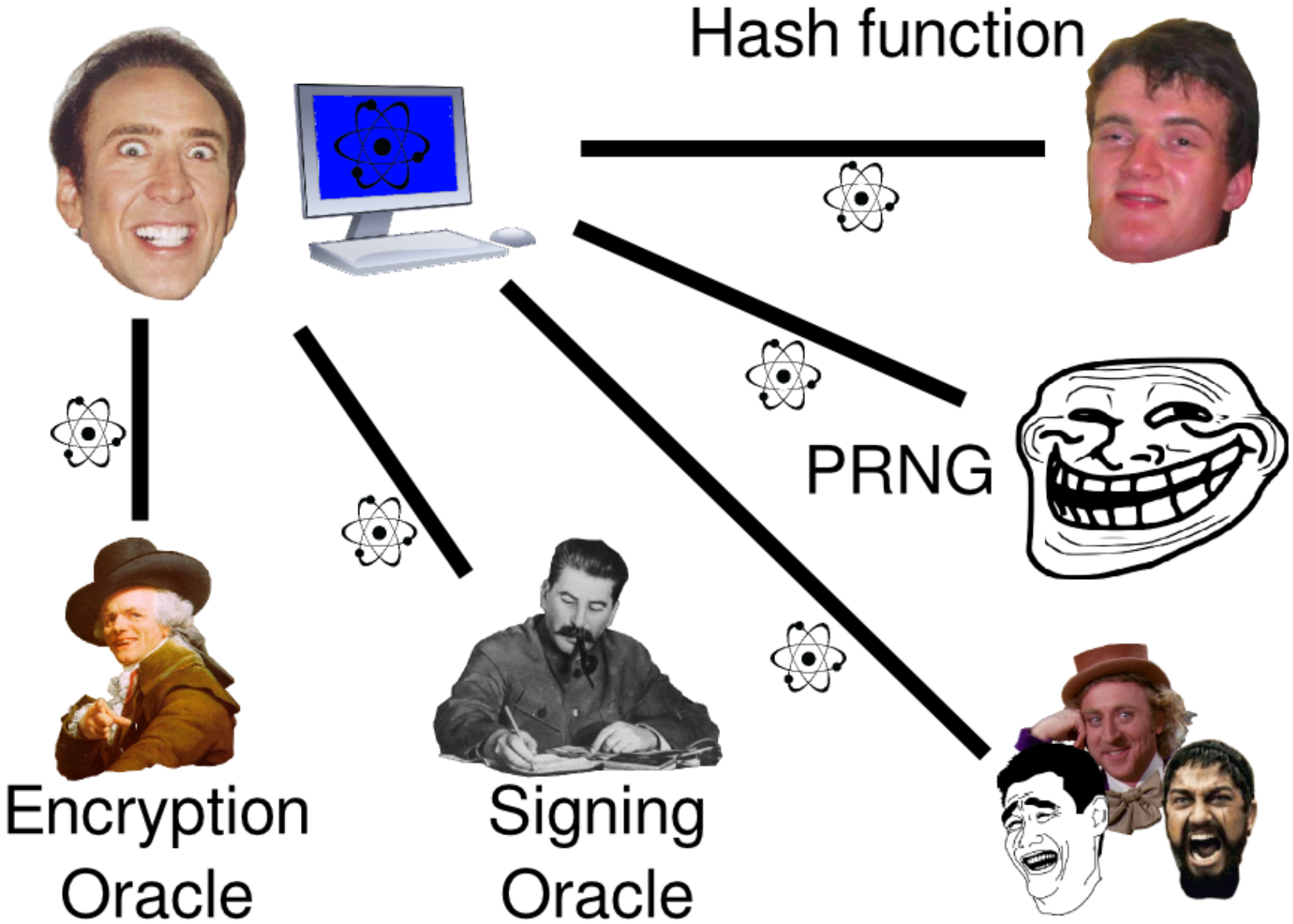

Example: Quantum CPA

CPA phase: $\mathcal{A}$ and $\mathcal{C}$ share a classical channel:

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What about encryption?

Fully Quantum Indistinguishability

In [BZ13][2]: fqIND phase: $\mathcal{A}$ and $\mathcal{C}$ share three quantum registers:

[2] D. Boneh, M. Zhandry: *'Secure Signatures and Chosen Ciphertext Security in a Quantum Computing World'*, CRYPTO 2013.

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- $\mathcal{A}$ must guess b.

[2] D. Boneh, M. Zhandry: *'Secure Signatures and Chosen Ciphertext Security in a Quantum Computing World'*, CRYPTO 2013.

Fully Quantum Indistinguishability

In [BZ13][2]: fqIND phase: $\mathcal{A}$ and $\mathcal{C}$ share three quantum registers:

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Theorem

qIND is unachievable (too strong).

(attack exploits entanglement between ciphertext and plaintext)

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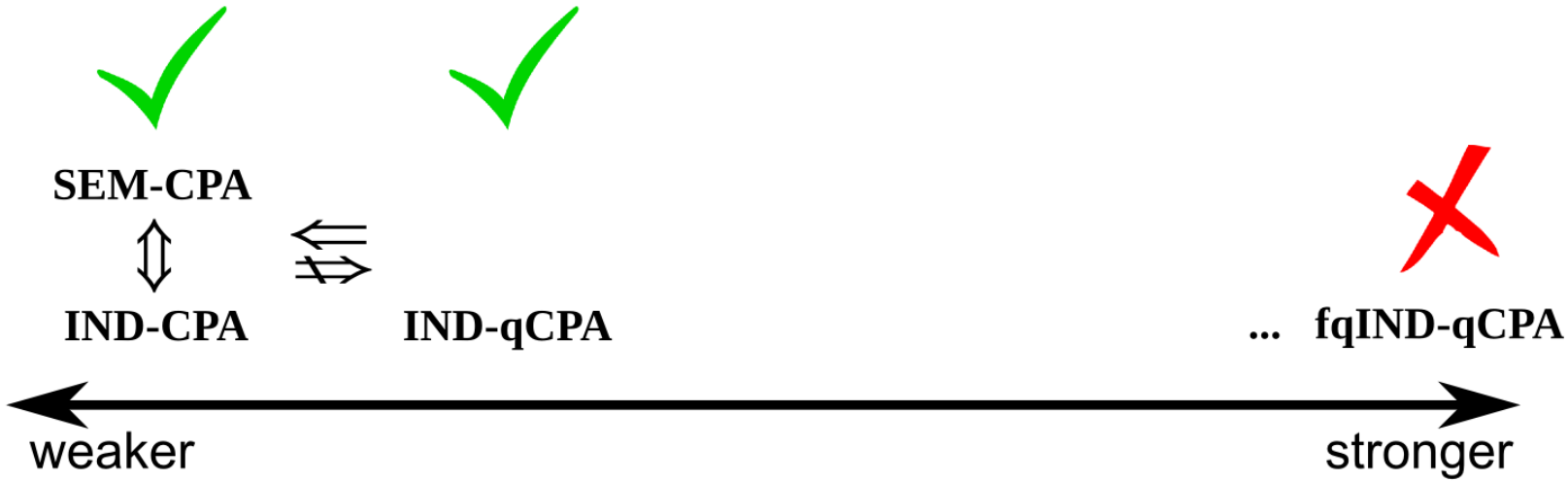
IND-qCPA is strictly stronger than classical IND-CPA.

The current situation



Unsatisfying...

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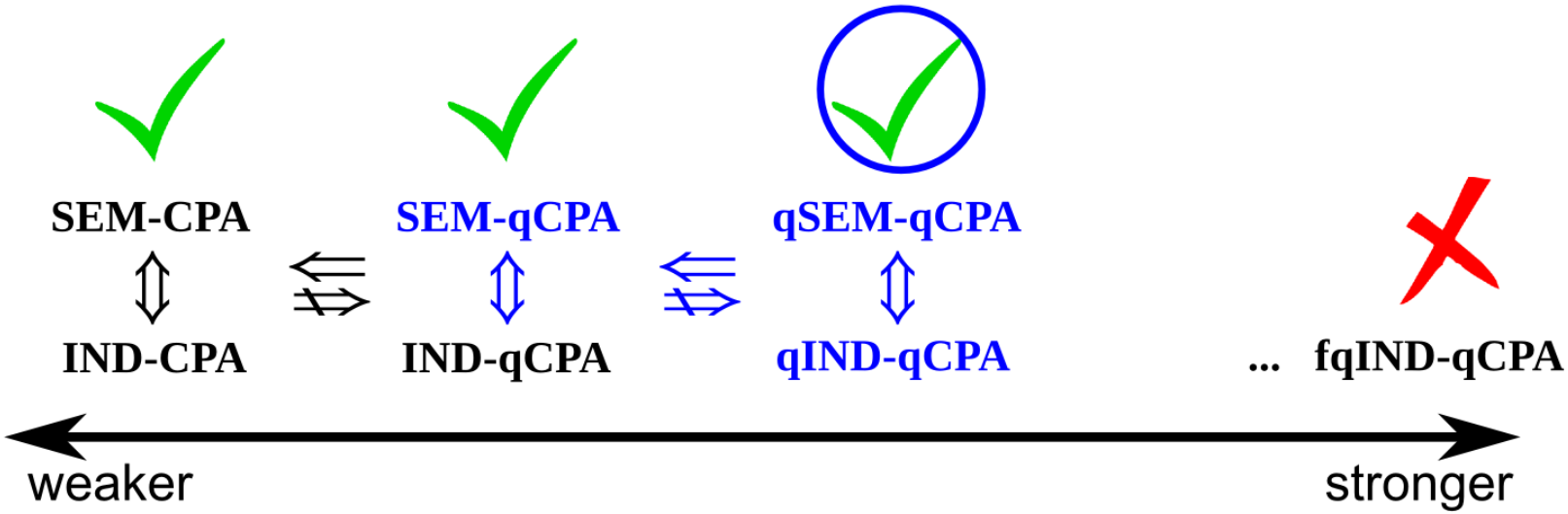

Our contribution!

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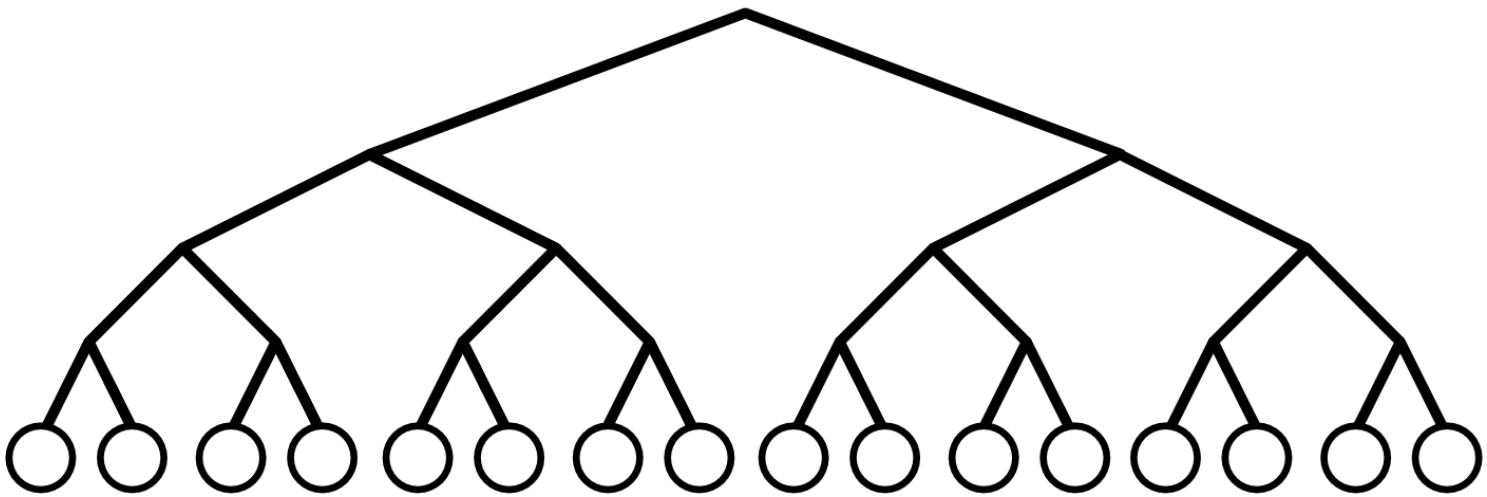

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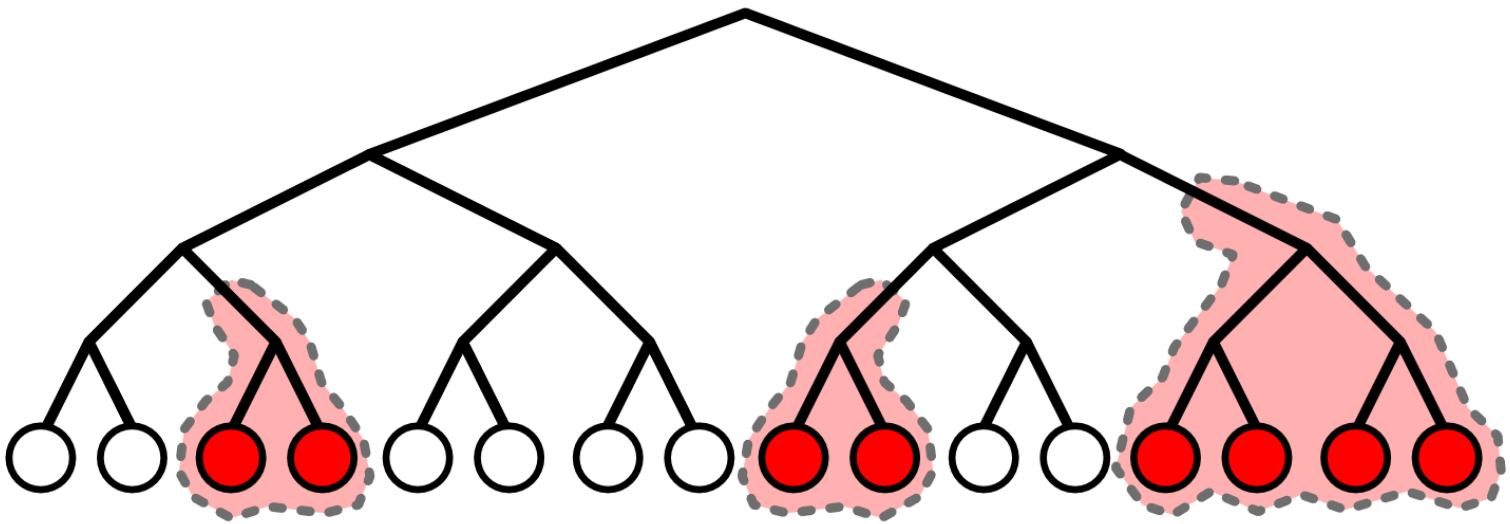

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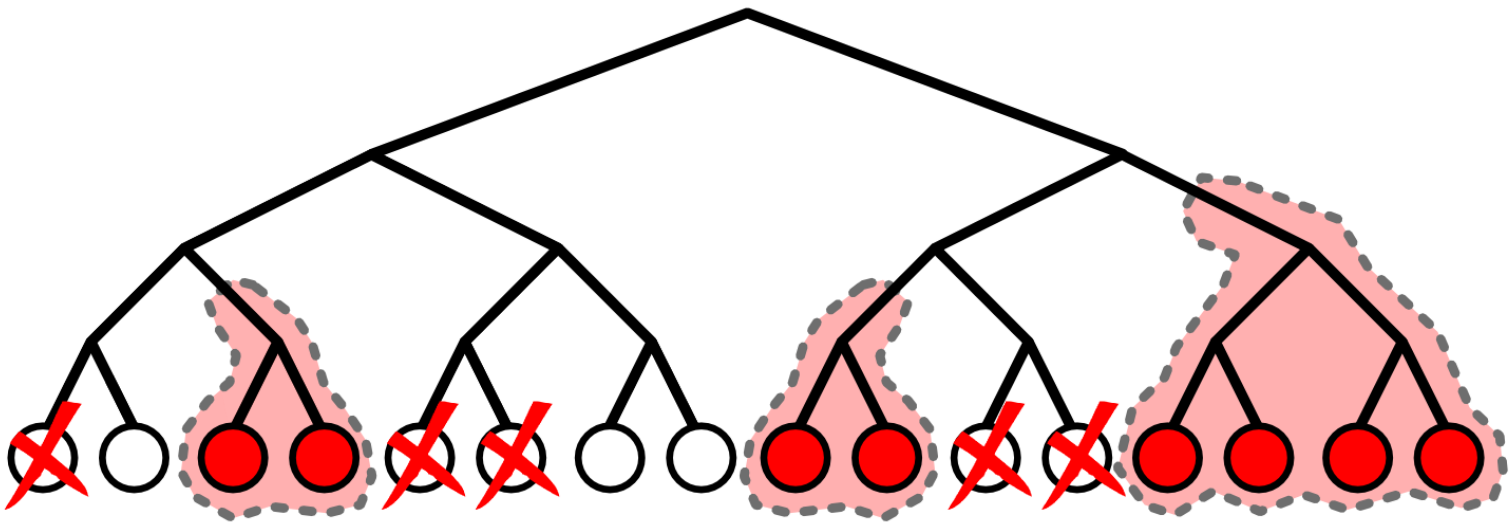

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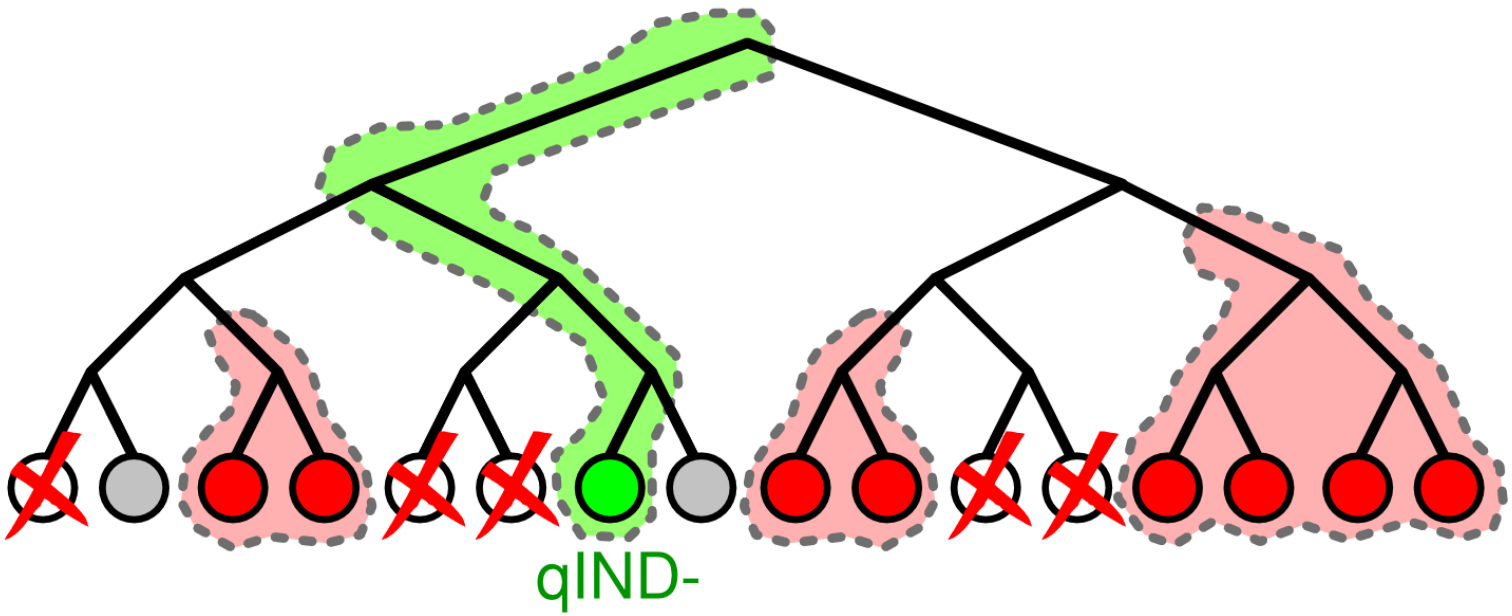
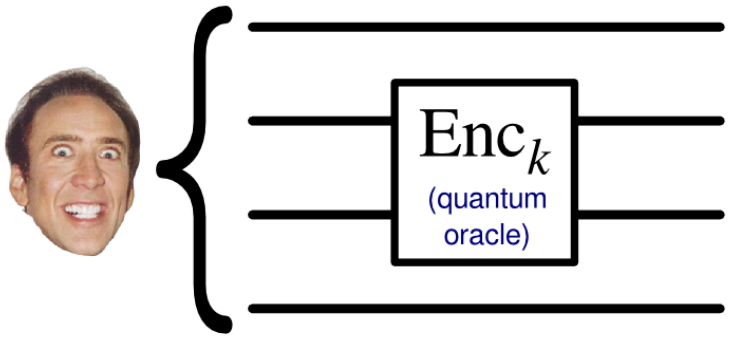


Model: $\mathcal{O}$ vs. $\mathcal{C}$

$(\mathcal{O})$



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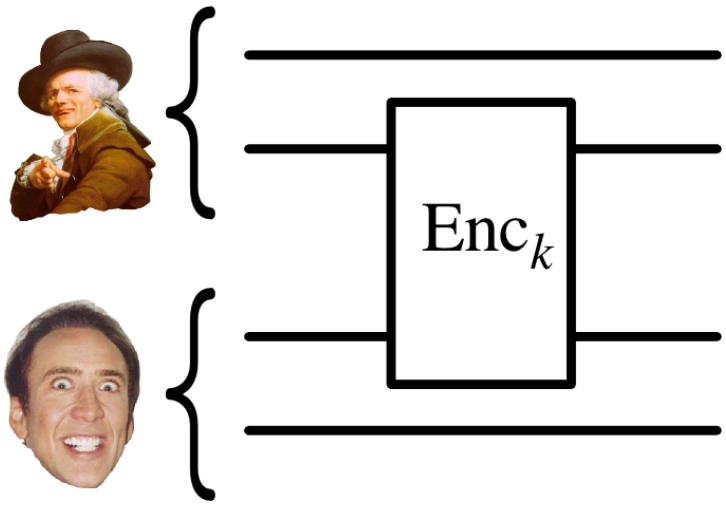
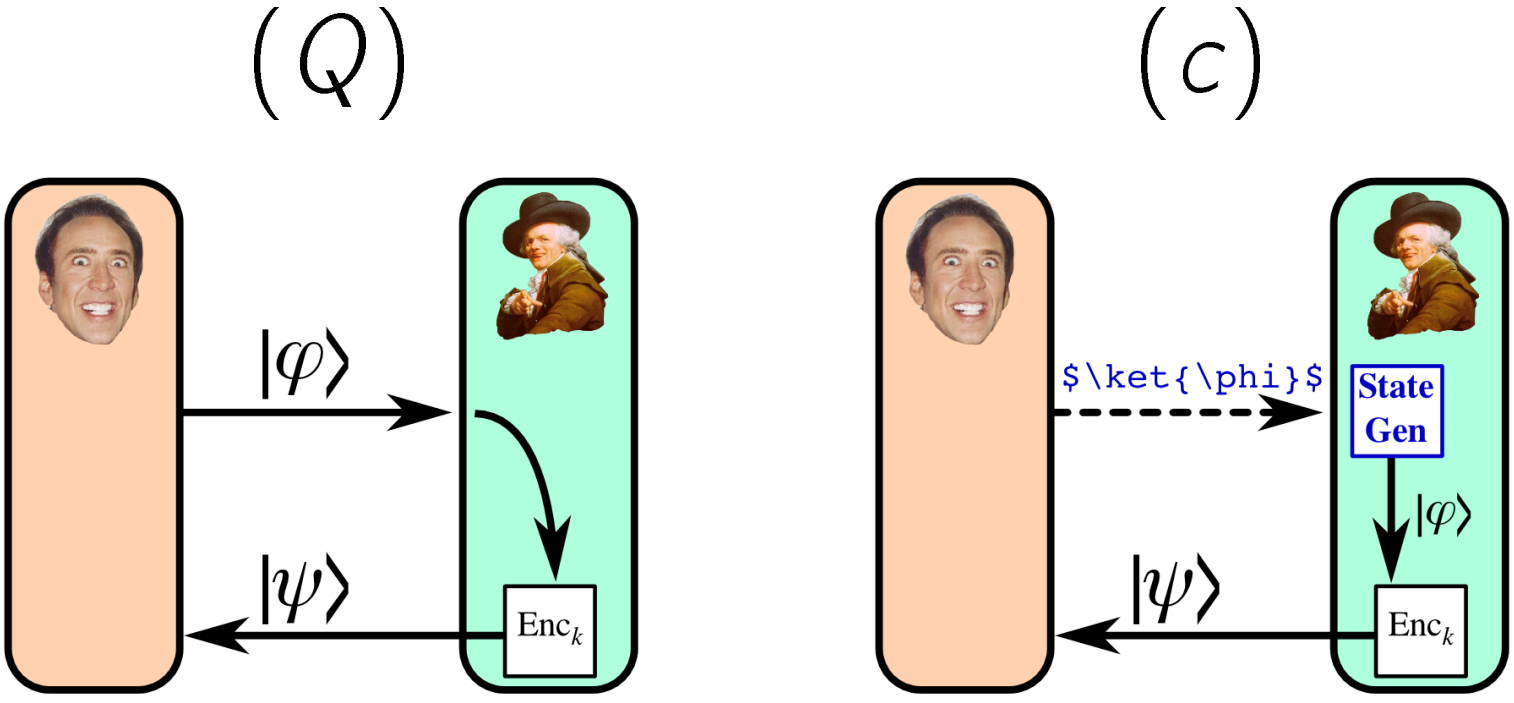


Model: Q vs. c


(Q)
(c)
$|\varphi\rangle$
$|\psi\rangle$
Enc_k
$\ket{\phi}$
State Gen
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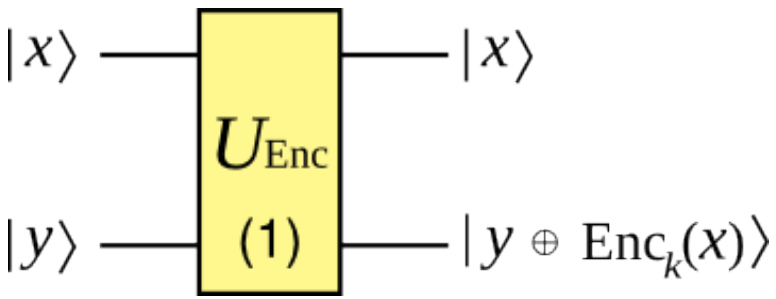
Enc_k

Model: Type-1 vs. Type-2 Transformations

(1)



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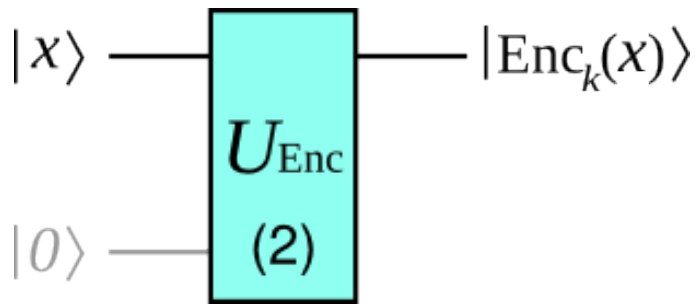

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what about quantum semantic security?

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Proof Idea:

'$\Rightarrow$': provide $\mathcal{S}$ with $\mathcal{A}$'s code through h, impersonate $\mathcal{C}$ and use IND to argue same prob.

'$\Leftarrow$': assume distinguisher $\mathcal{A}$, choose constant h, then no $\mathcal{S}$ can infere anything w/o ciphertext.

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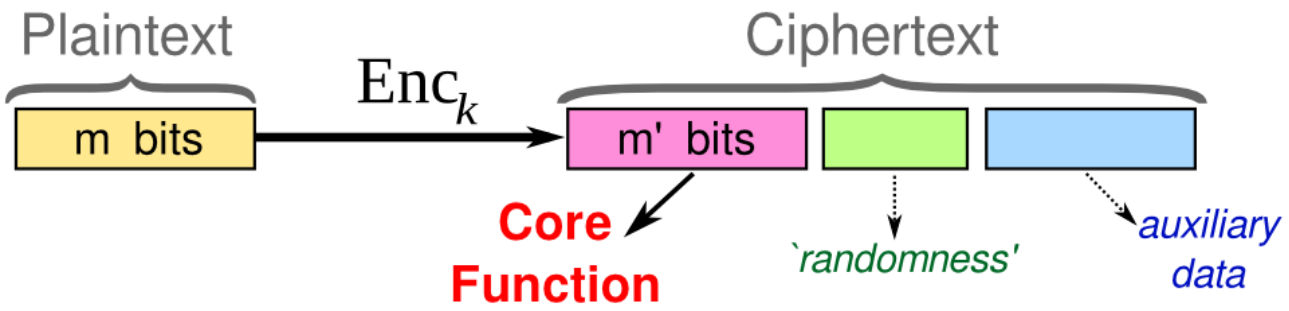

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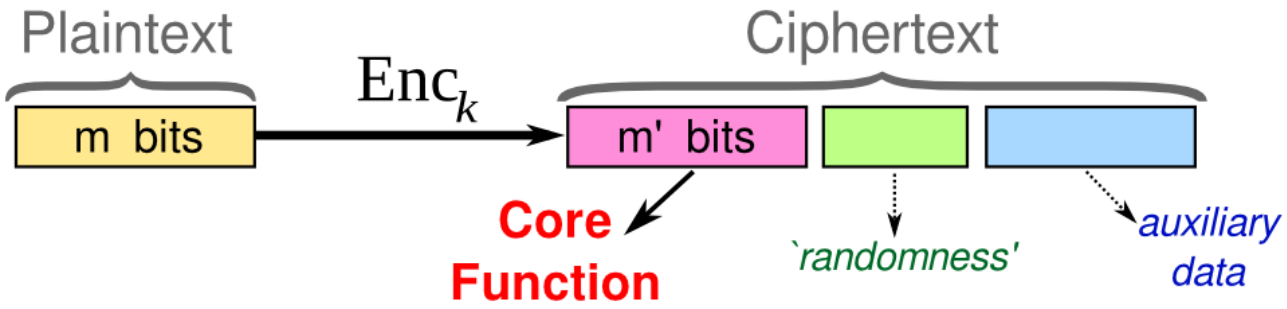

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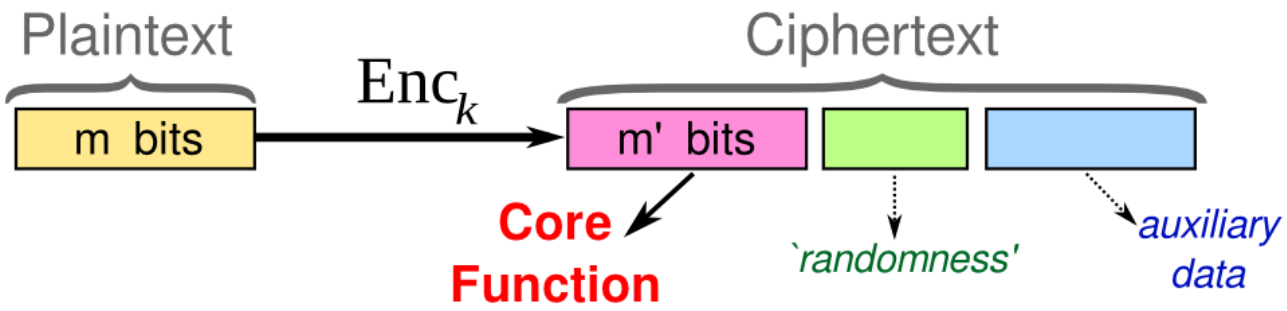

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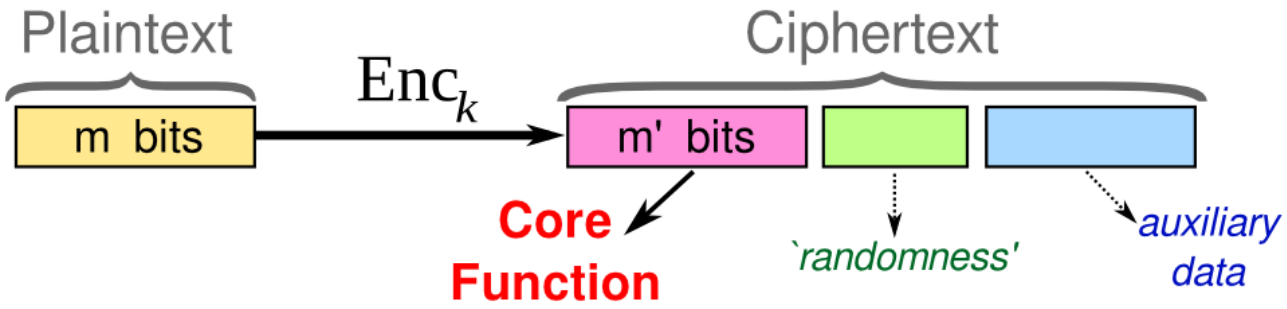

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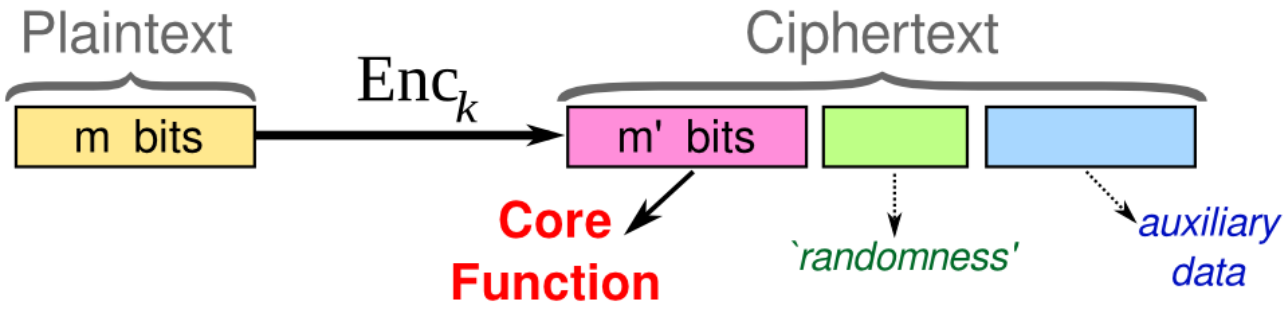

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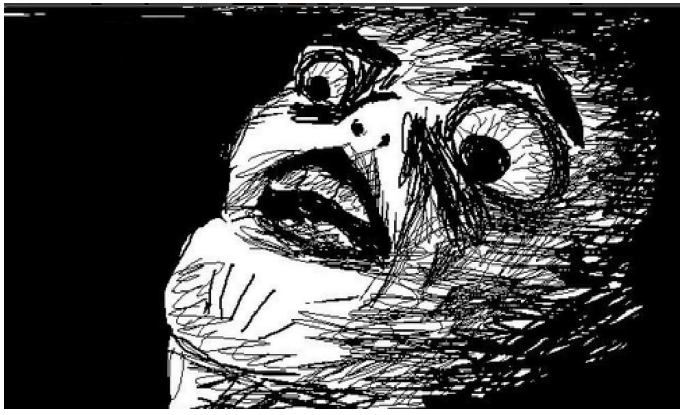

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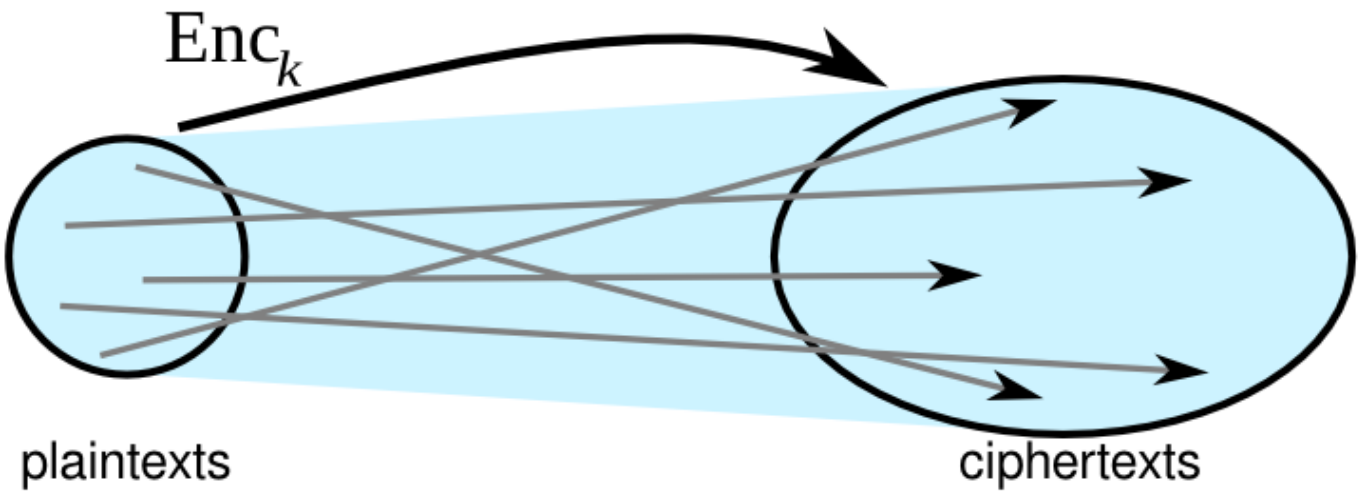

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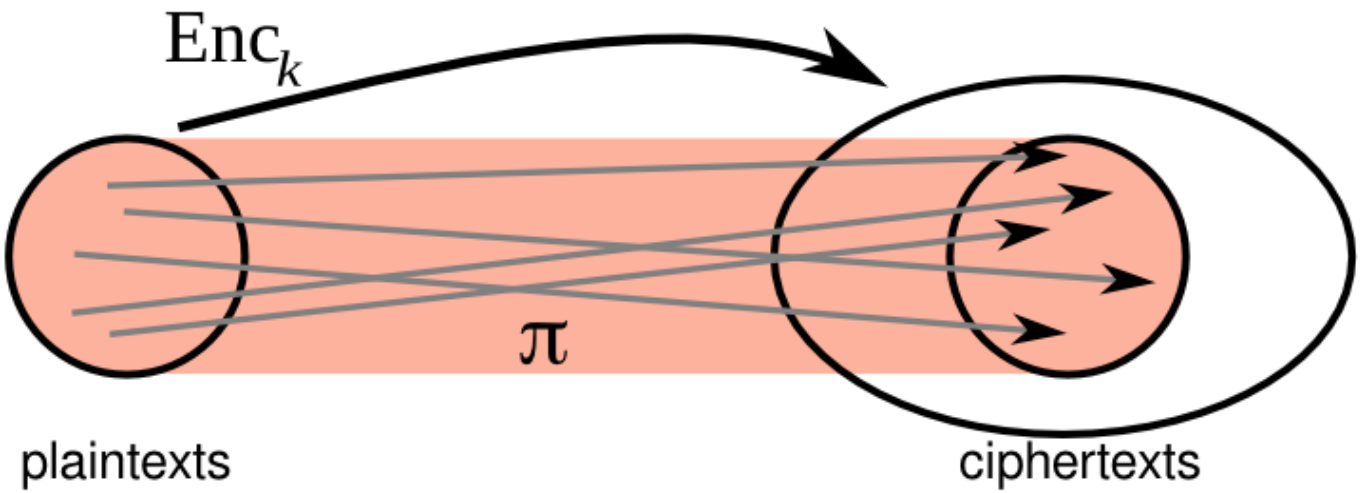
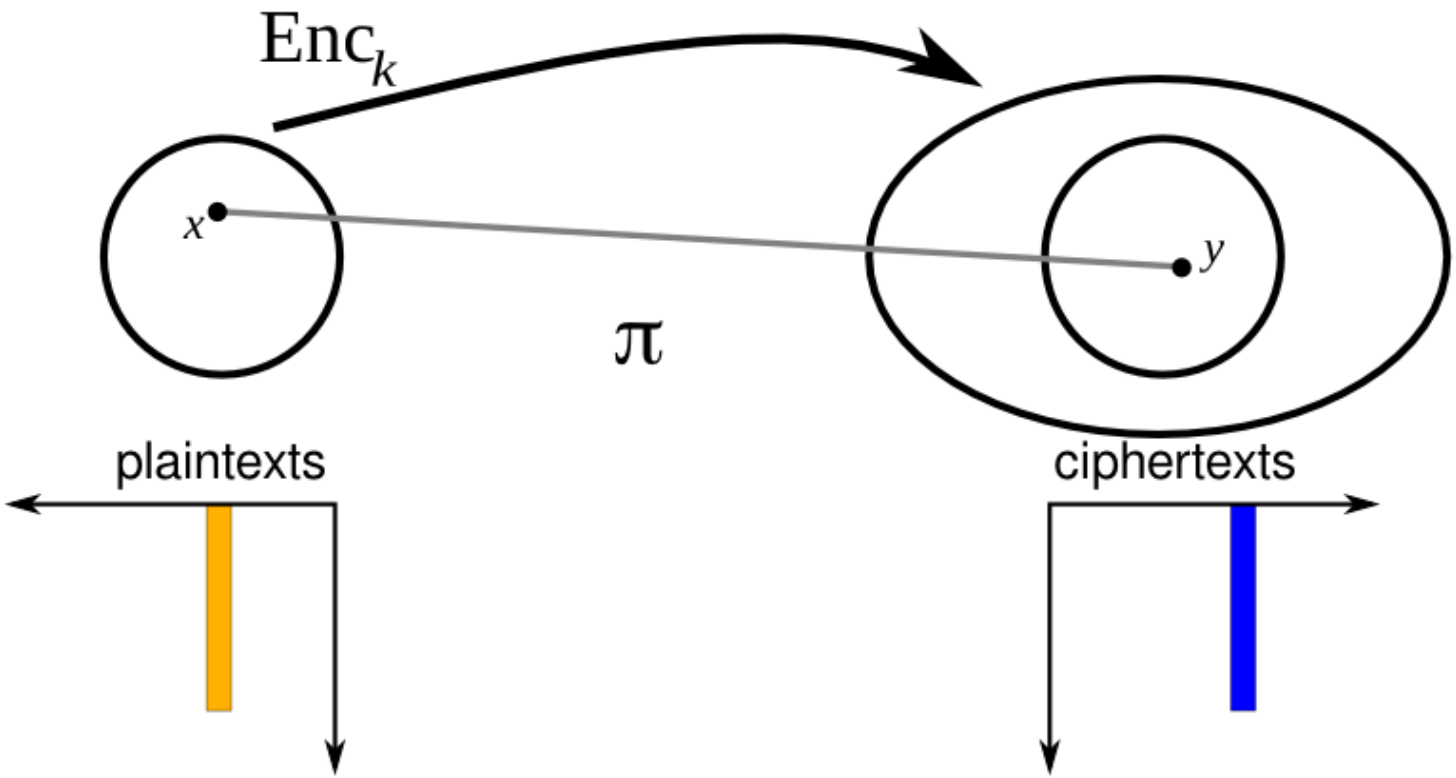


QLP cipher

Core Function = permutation π

The Attack



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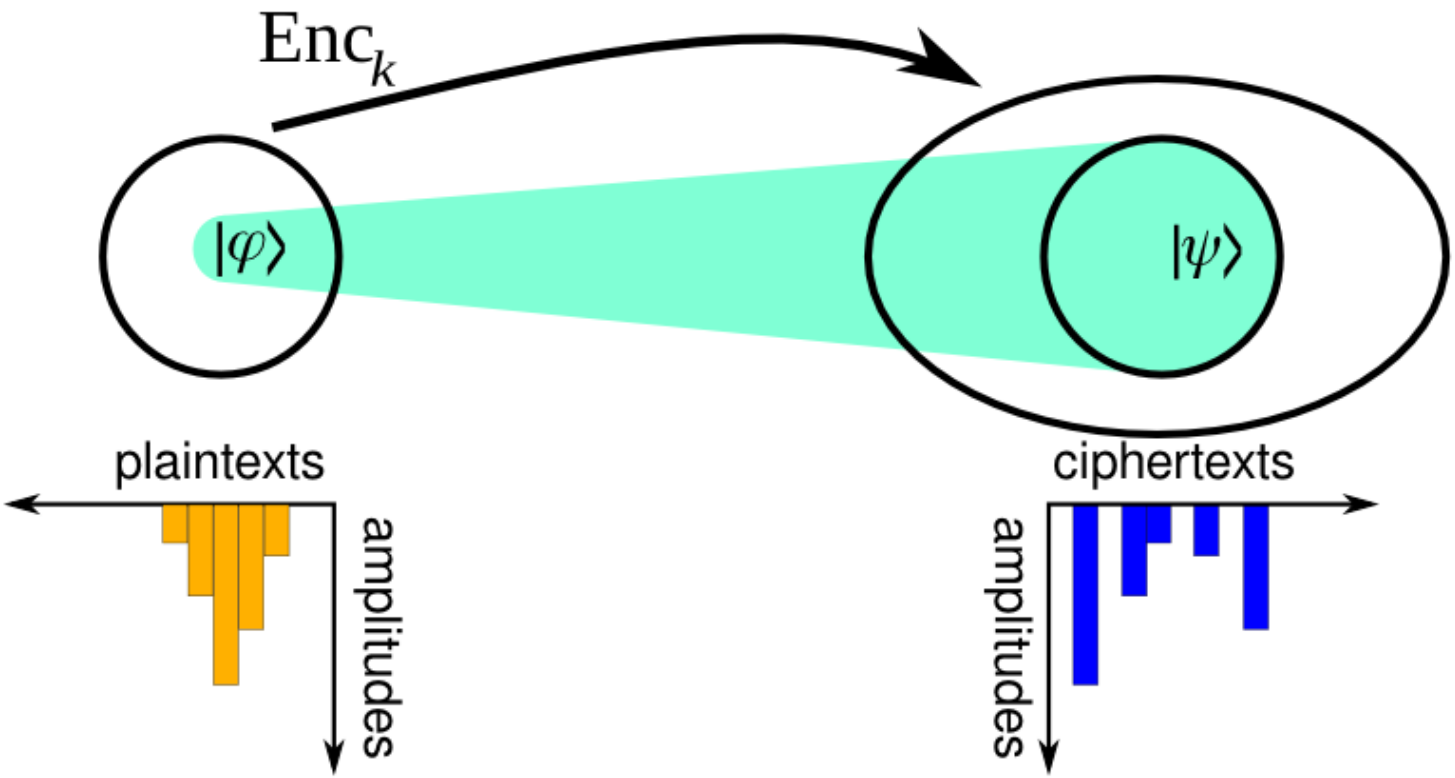
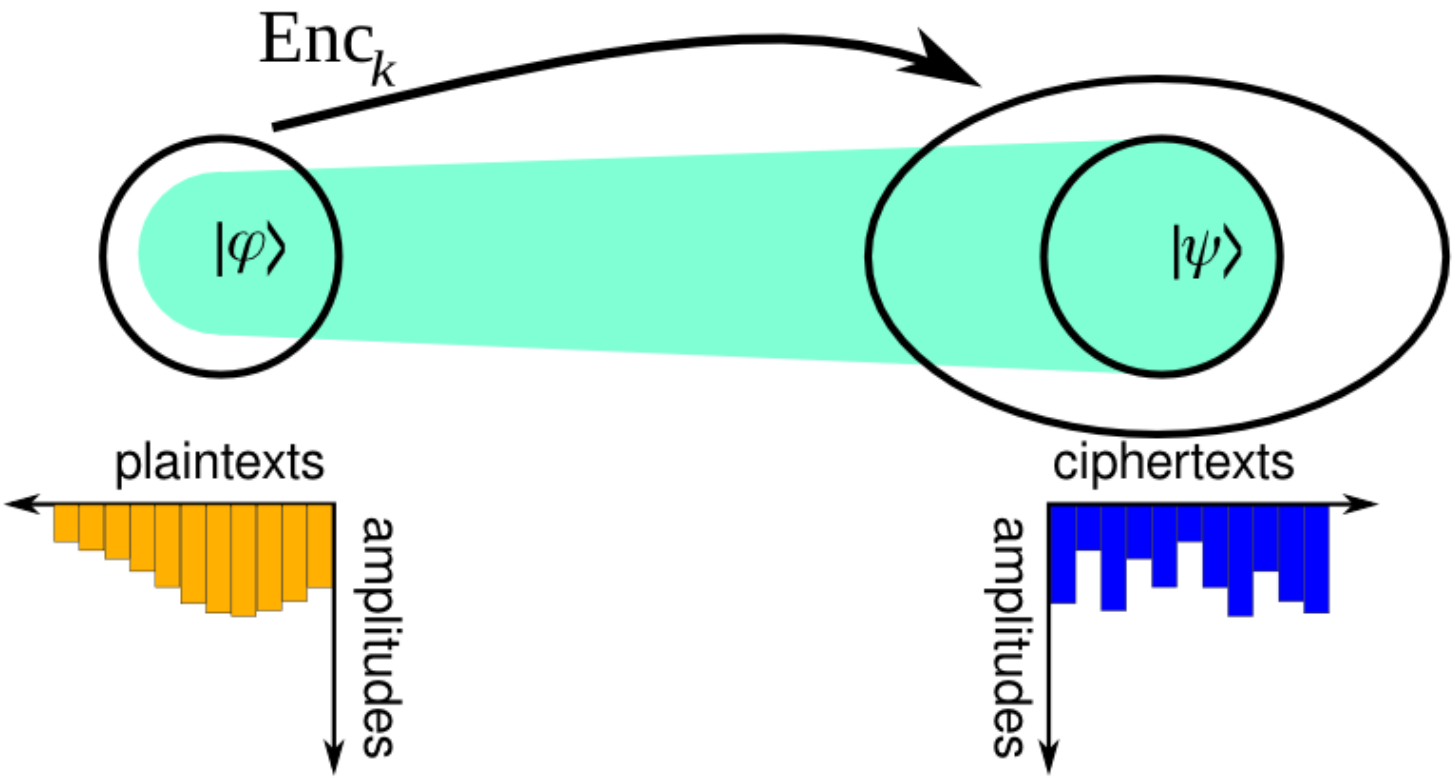

Enc_k
$|\varphi\rangle$
$|\psi\rangle$
plaintexts
amplitudes
ciphertexts
amplitudes

The Attack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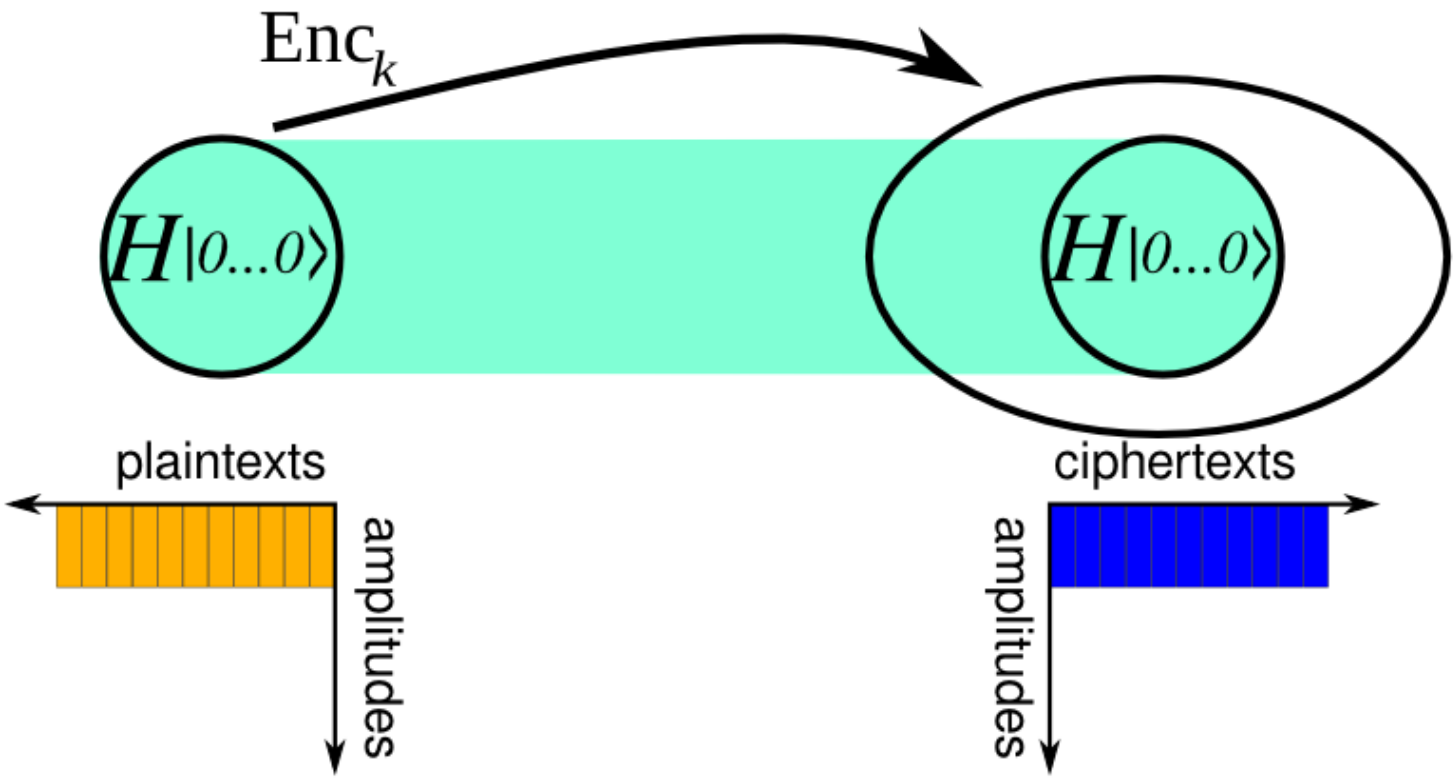
Enc_k
$H|0\ldots0\rangle$
$H|0\ldots0\rangle$
plaintexts
amplitudes
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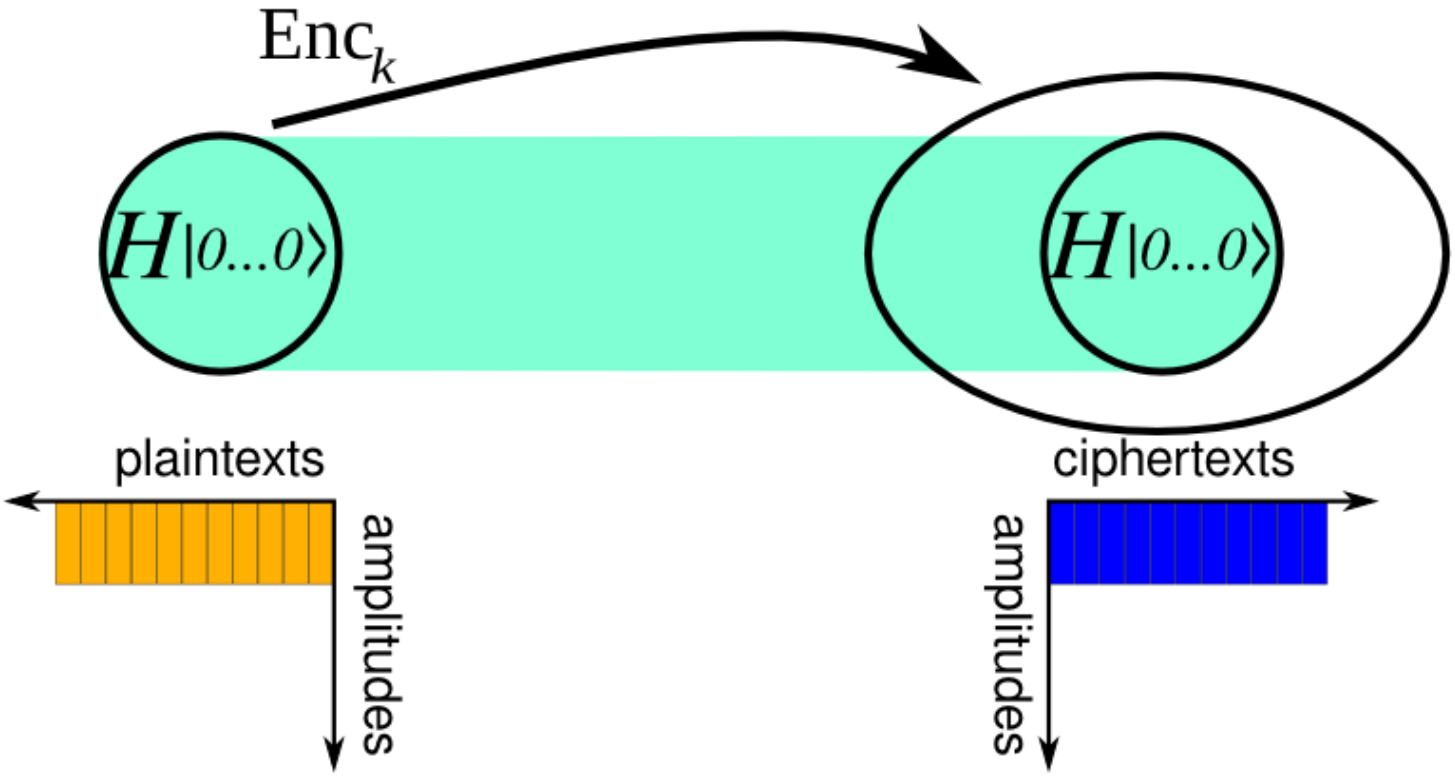
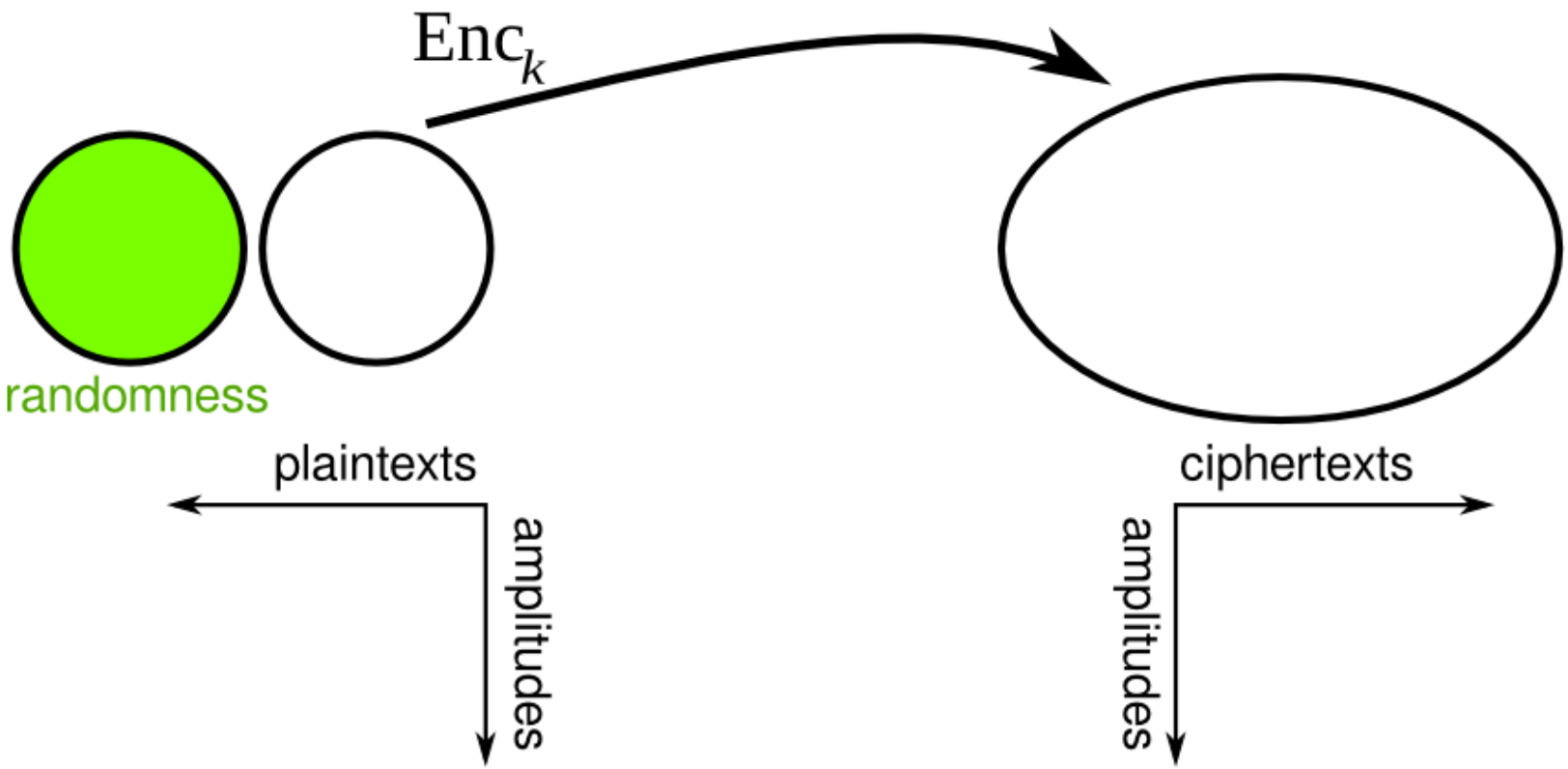
The Attack



$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \xrightarrow{\mathsf{Enc}_k} \frac{1}{\sqrt{2}}|\pi(0)\rangle + \frac{1}{\sqrt{2}}|\pi(1)\rangle = |+\rangle$$

Easy to distinguish!

The Solution



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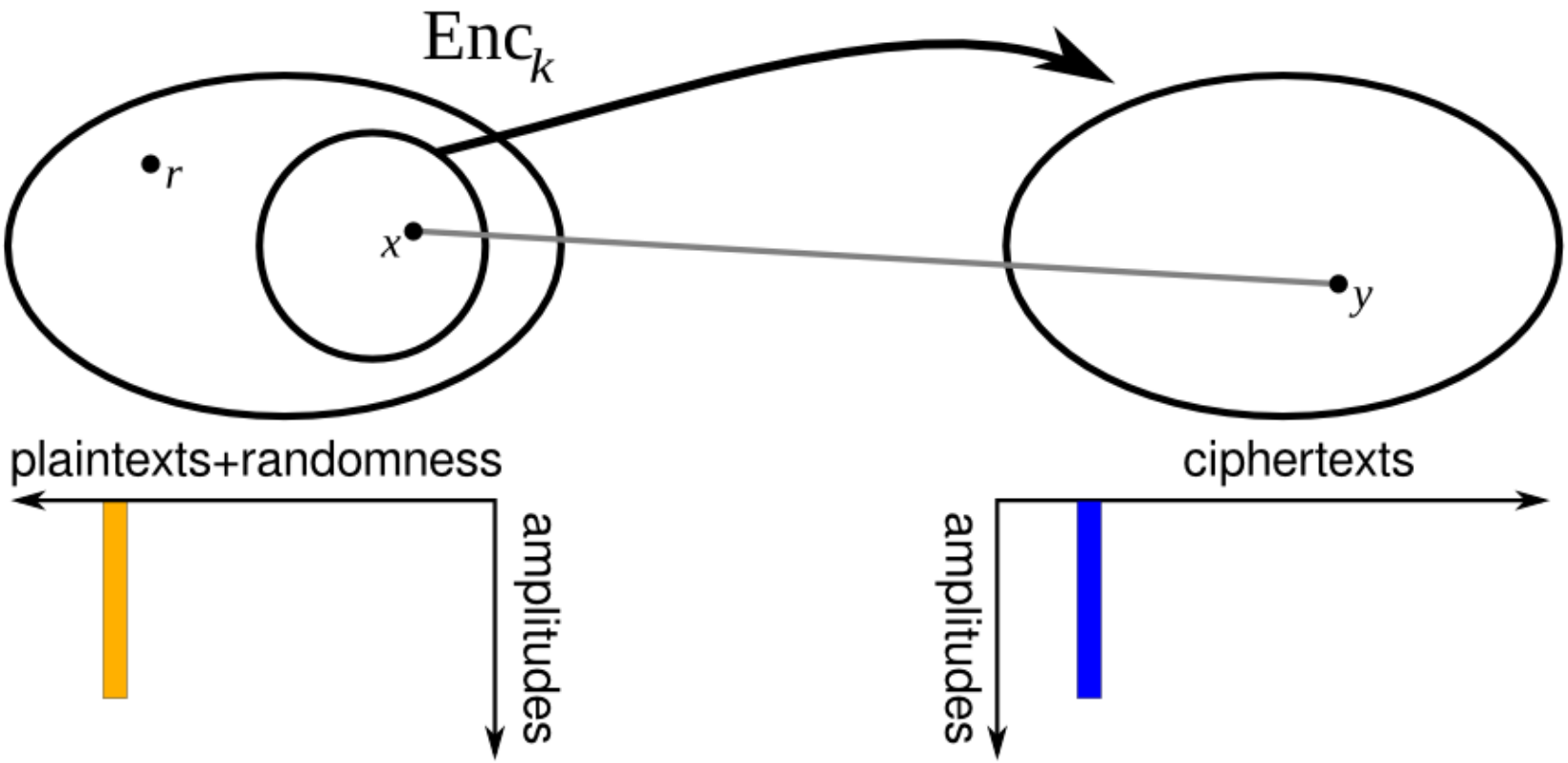
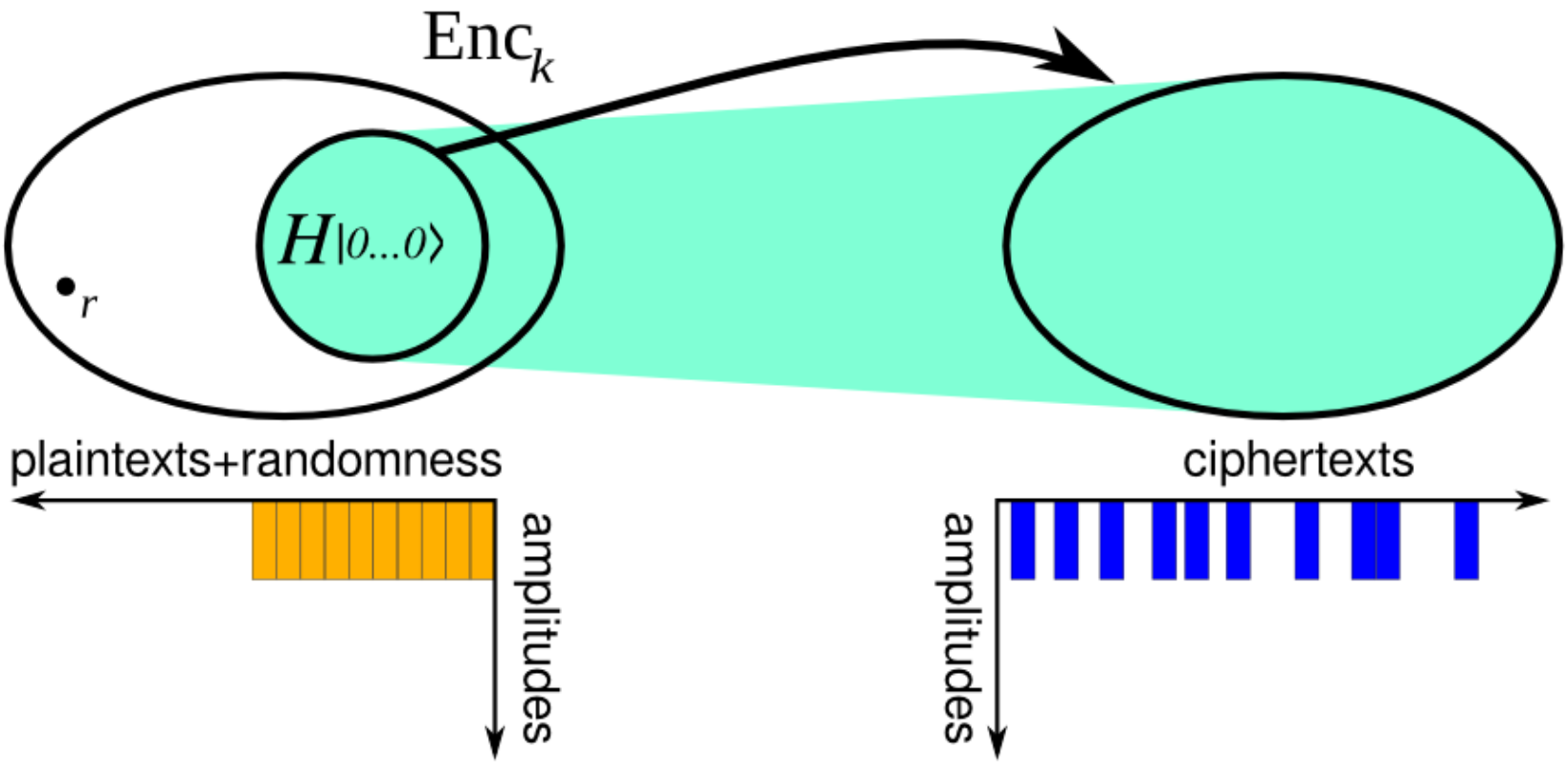


The Solution



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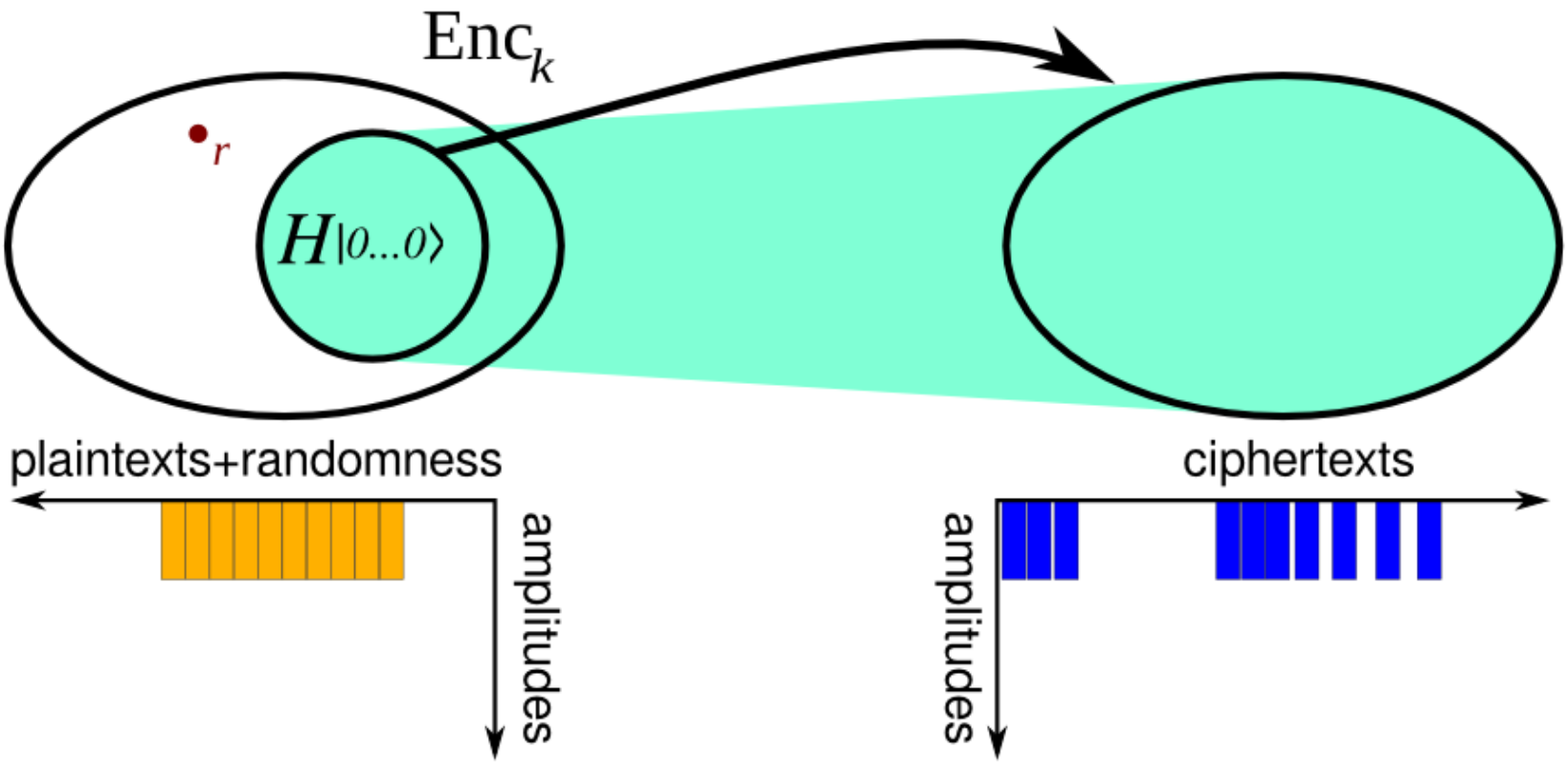
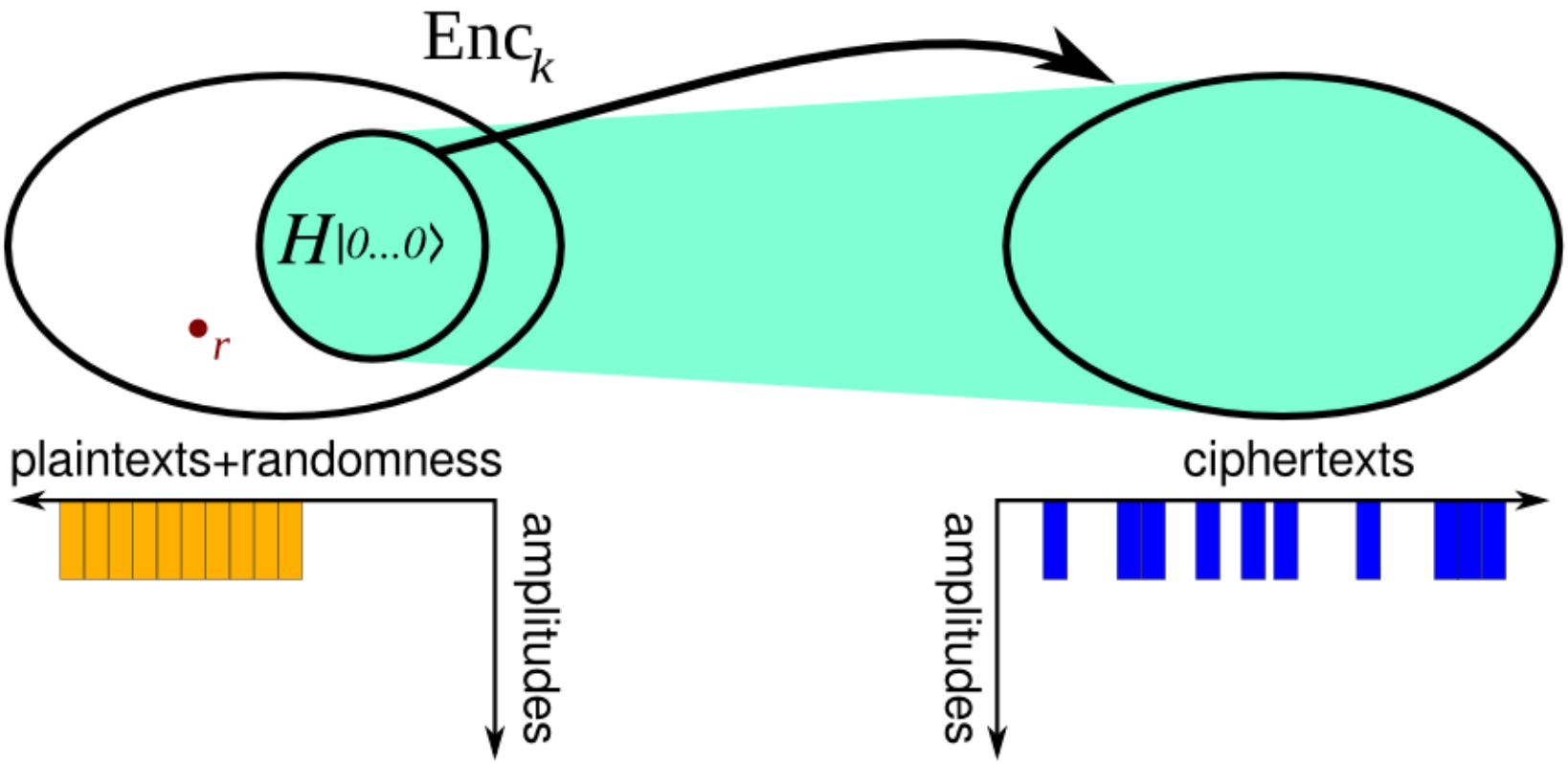


The Solution



Secure Construction

Π family of quantum-secure pseudorandom permutations (QPRP).

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Construction

- Generate key: sample $(\pi, \pi^{-1}) \leftarrow \Pi$;
- Encrypt message x: pad with n bits of randomness r and set $y = \pi(r \| x)$;
- Decrypt y: truncate the first n bits of $\pi^{-1}(y)$.

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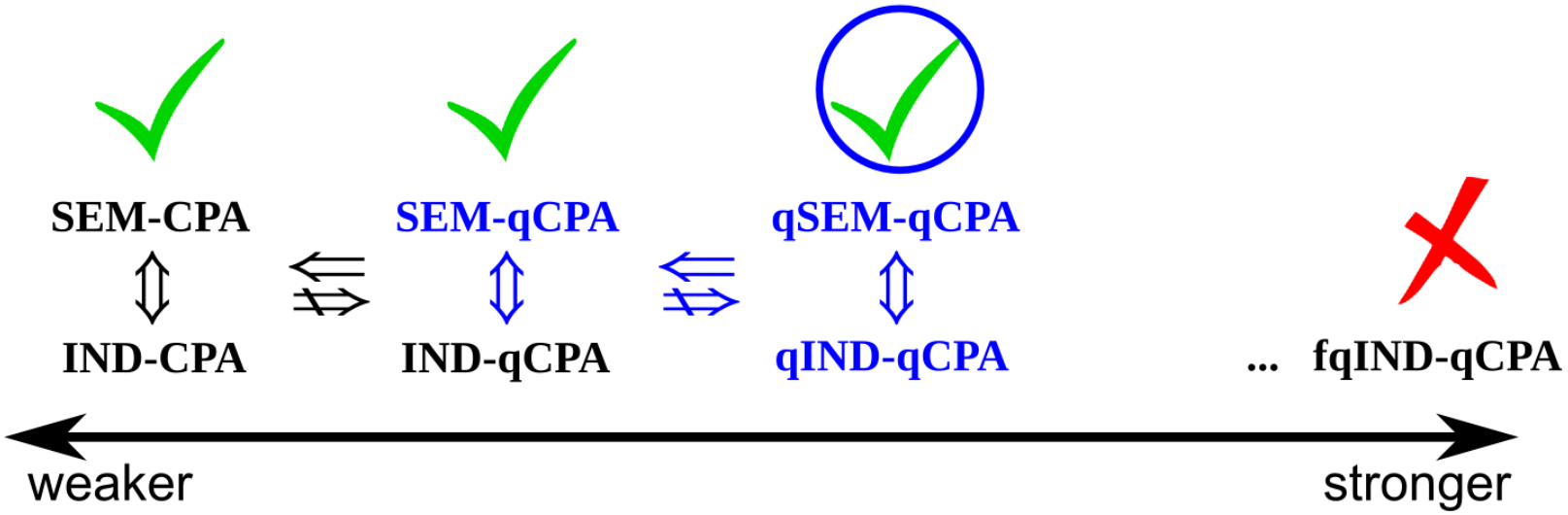
Theorem

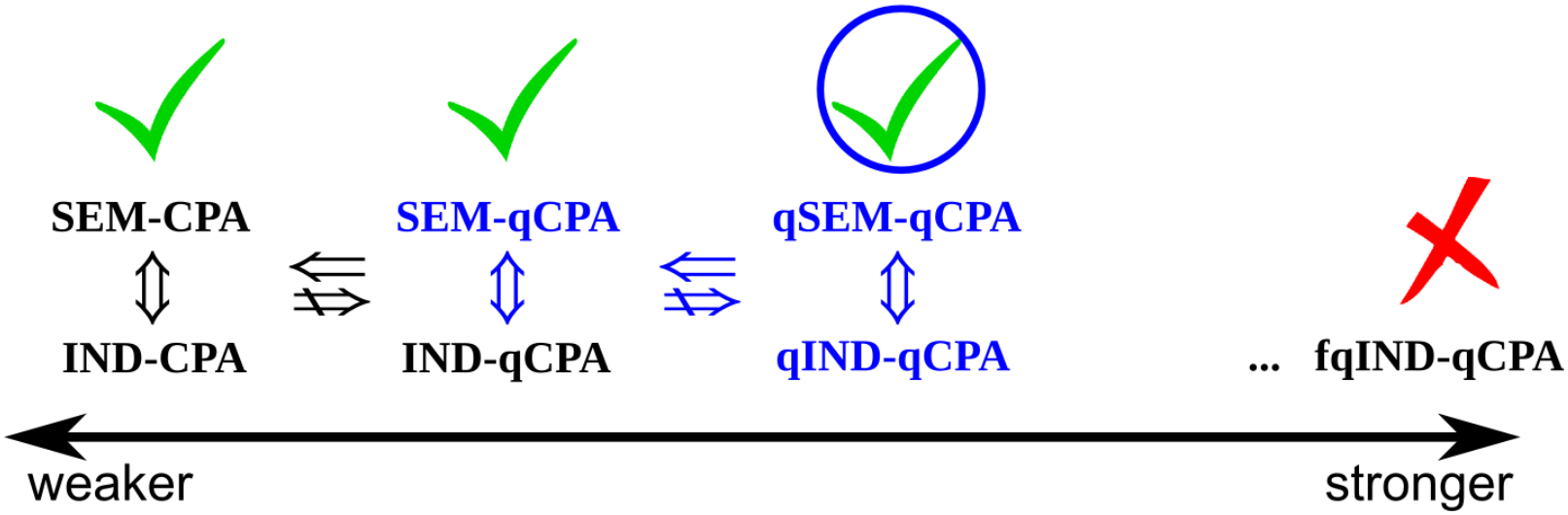
The above scheme is qIND-qCPA secure.

(Idea of proof: show that for every two plaintext states $|\phi_0\rangle, |\phi_1\rangle$, the trace distance of the states ρ_0, ρ_1 obtained by considering their encryption under a mixture of every possible key is negligible)

Conclusions



Conclusions



Future directions:

- public-key encryption;
- CCA security;
- slightly different models of qIND or qSEM;
- superposition of keys/randomness;
- patch IND-qCPA $\Rightarrow$ qIND-qCPA;
- 'fully' quantum scenario (ongoing work).

End of this talk

Thanks for your attention!

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Cryptography & Complexity Theory
Technische Universität Darmstadt
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BZ Attack

(example for 1-bit messages, with normalization amplitudes omitted)

$\mathcal{A}$ initializes register to: $H|0\rangle \otimes |0\rangle \otimes |0\rangle = \sum_x |x, 0, 0\rangle$ and then calls the encryption oracle with unknown bit b. Now:

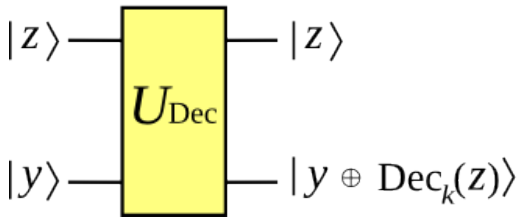
- if $b = 0$, the state becomes: $\sum_x |x, 0, \mathrm{Enc}(x)\rangle$ (notice entanglement between 1^{st} and 3^{rd} registers);
- if $b = 1$ instead, the state becomes: $\sum_x |x, 0, \mathrm{Enc}(0)\rangle = H|0\rangle \otimes |0\rangle \otimes |\mathrm{Enc}(0)\rangle$.

Then $\mathcal{A}$ applies a Hadamard on the 1^{st} register and measures:

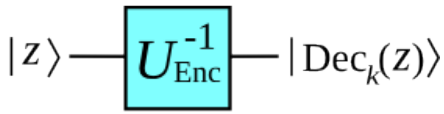
- if $b = 0$, the Hadamard maps the state to a complete mixture, and the measurement outcome is random;
- if $b = 1$ instead, the first register is: $H^2|0\rangle = |0\rangle$, and the outcome is 0.

Equivalence between Type-1 and Type-2

Type-1
Decryption Oracle



Type-2
Decryption Oracle



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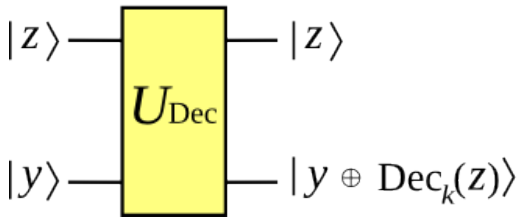
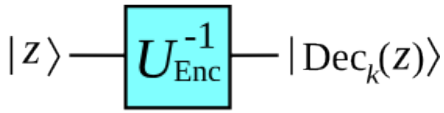
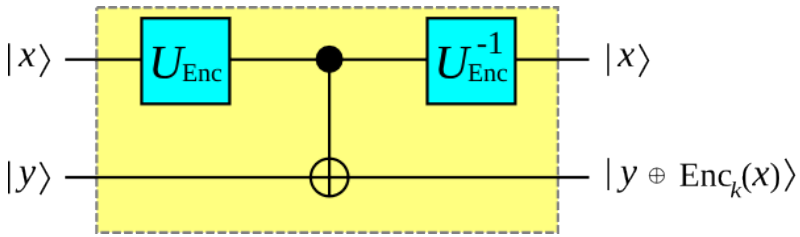
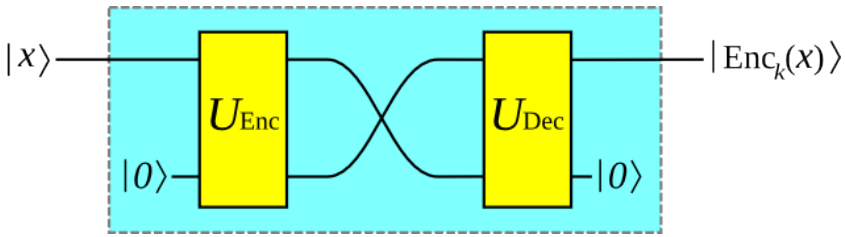


Type-2
Decryption Oracle







qSEM $\Rightarrow$ qIND

By contradiction: let $\mathcal{A}$ be an efficient qIND distinguisher. We show that there exists an efficient $\mathcal{A}'$ for qSEM which does not admit simulator.
$\mathcal{A}'$ invokes $\mathcal{A}$, which starts a qIND challenge query consisting of two classical descriptions s_0, s_1 of states $|\phi_0\rangle, |\phi_1\rangle$.
$\mathcal{A}'$ records this template, then prepare his own qSEM challenge template consisting of:

- as distribution X, the uniform distribution over $\{s_0, s_1\}$;
- as advice function h, a constant function (not depending on s_0, s_1);
- as target function f, the *identity function* $f(x) = x$.

$\mathcal{A}'$ receives $\mathcal{C}$'s response, forwards the ciphertext to $\mathcal{A}$, and observes output.
Since $\mathcal{A}$ recovers b with non-negligible probability, $\mathcal{A}'$ can then reconstruct the correct $|\phi_b\rangle$ (having recorded its description) and compute the output reduced state ρ_f.
Any simulator $\mathcal{S}$, on the other hand, only receives a constant function h, and then cannot do better than guessing.

qSEM $\Leftarrow$ qIND

Let $\mathcal{A}$ be any QPT adversary against qSEM. Then its circuit has a short classical representation ξ.

Then here is a simulator $\mathcal{S}$ with the same success probability:

1. $\mathcal{S}$ receives ξ as nonuniform advice (this is allowed);
2. then $\mathcal{S}$ implements and run $\mathcal{A}$ through ξ;
3. when $\mathcal{A}$ produces a qSEM challenge template (X, h, f), $\mathcal{S}$ forwards it to $\mathcal{C}$;
4. when $\mathcal{C}$ replies with its advice function, $\mathcal{S}$ forwards it to $\mathcal{A}$, together with the encryption of a bogus state;
5. finally, $\mathcal{S}$ outputs whatever $\mathcal{A}$ does.

The presence of the bogus encryption state instead of the right one does not affect $\mathcal{A}$'s success probability. In fact, if this were the case, we could turn $\mathcal{S}$ into an efficient distinguisher against qIND.

The ‘Frozen Smartcard’ Example

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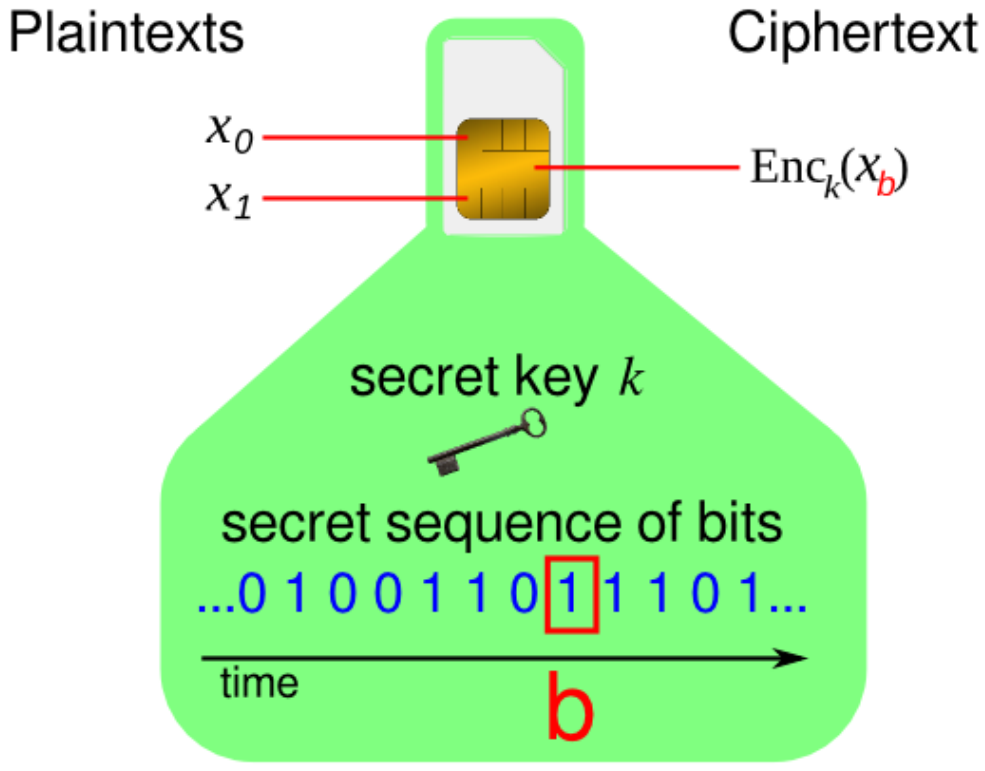

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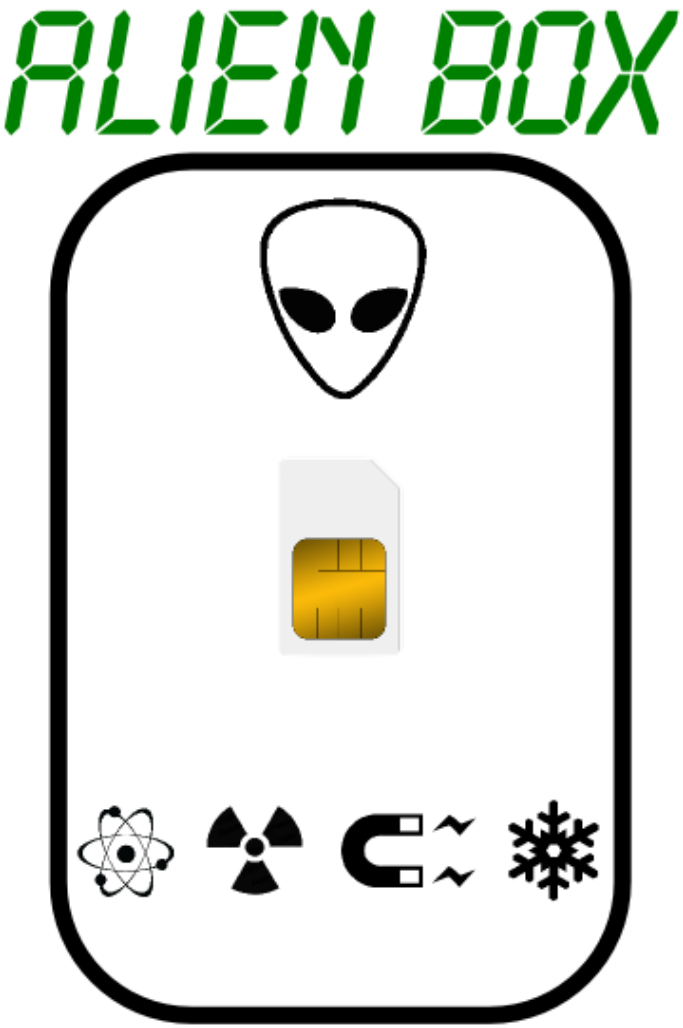

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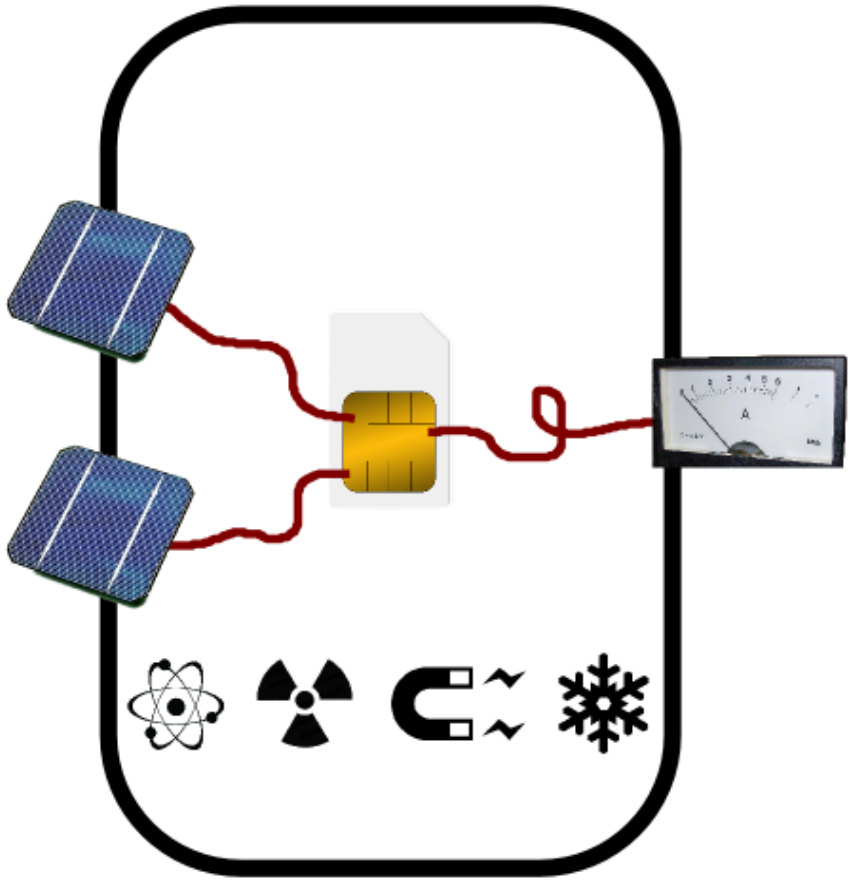

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